Exercise 5-Solutions
TMA4255
Applied Statistics

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1 Problem 1

- $H_0: \ \mu = 5.5$
- $H_1: \ \mu < 5.5$

  $\alpha = 0.05$

  Critical region: $z < -z_\alpha$, where $z_\alpha = 1.645$ and $z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$

  Computations: $\bar{x} = 5.23$, $\sigma = 0.24$, and hence $z = \frac{5.23-5.5}{0.24/\sqrt{64}} = -9$

  Decision: reject H_0.

The P-value corresponding to $z = -9$ is $P = P(Z < 9)$ that is lower than α. This is another evidence to reject H_0.

2 Problem 2

- $H_0: \ \mu = 5.5$
- $H_1: \ \mu < 5.5$

With significance level $\alpha = 0.05$ and variance known.

Power test $= 0.90 = 1 - \beta$, hence $\beta = 0.10$.

We know from the theory that

$$n \approx \frac{(z_\alpha + z_\beta)^2 \sigma^2}{\delta} = \frac{(1.282 + 1.645)^2 0.24^2}{0.3^2} = 5.45,$$

where $\delta = \mu - \mu_0$.
So we choose $n = 6$.

3 Problem 3

In this case we want to test the hypothesis

- $H_0: \; p = 0.6$
- $H_1: \; p \neq 0.6$

  $\alpha = 0.05$

  Test statistics: Binomial variable X with $p = 0.6$ and $n = 200$

  Computations: since the sample size n is large we prefer to approximate X with a Gaussian $N(np_0, np_0q_0) = N(120, 48)$.
  $x = 110$, so the P-value is given by

$$2P(X < 120) = 2P(\frac{X - 120}{\sqrt{48}} < \frac{110 - 120}{\sqrt{48}}) = 0.095.$$

  The P-value is higher than α, so H_0 is not rejected.

4 Problem 4

	Non-smokers	**Moderate smokers**	**Heavy smokers**	**Total**
Hypertension	21	36	30	87
No Hypertension	48	26	19	93
Total	69	62	49	180

Let's start computing the expected frequency:

$$\text{expected frequency} = \frac{(\text{column total}) \times (\text{row total})}{\text{grand total}}$$

the computed expected frequencies are recorded in parenthesis inside the table.

	Non-smokers	**Moderate smokers**	**Heavy smokers**	**Total**
Hypertension	21 (33.35)	36 (29.9)	30 (23.68)	87
No Hypertension	48 (35.65)	26 (32.03)	19 (25.31)	93
Total	69	62	49	180

and the degree of freedom is given by

$$\nu = (r-1)(c-1) = 2.$$

Then to test the independence we compute:

$$\chi^2 = \sum_i \frac{(o_i - e_i)^2}{e_i} = 14.4$$

Since $\chi^2_{0.05} = 5.991$, we have that $\chi^2 > \chi^2_\alpha$, hence we reject the null hypothesis of independence.