NTNU
Norwegian University of
Science and Technology

Department of Mathematical Sciences

# Examination paper for **TMA4255 Applied statistics**

**Academic contact during examination:** Anna Marie Holand

**Phone:** 951 38 038

**Examination date:** August 2016

**Examination time (from–to):**

**Permitted examination support material:** C: Yellow, stamped A4 sheet with your own handwritten notes, Tabeller og formler i statistikk (Tapir forlag/Fagbokforlaget). Specified calculator.

**Other information:**

- In outputs from MINITAB comma is used as decimal separator.
- Significance level 5% should be used unless a different level is specified.
- All answers need to be justified.

**Language:** English

**Number of pages:** 10

**Number of pages enclosed:** 0

**Checked by:**

Date Signature

## Problem 1 Treatments for stress reduction

A study was conducted comparing three different treatments for stress reduction. In the study 30 participants were randomly divided into three different treatment groups, group A was given a mental treatment, group B was given a physical training treatment, and group C was given a medication treatment for stress reduction. A score in the range between 1 and 5 was given after the completion of the treatments. The score represents how effective the treatments were at reducing participant's stress levels, with higher numbers indicating high effectiveness.

Data and summary statistics for the treatment scores for the 3 different groups are given in Table 1 and Table 2, respectively.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Group A | 2 | 2 | 3 | 4 | 4 | 5 | 3 | 4 | 4 | 4 |
| Group B | 4 | 4 | 3 | 5 | 4 | 1 | 1 | 2 | 3 | 3 |
| Group C | 1 | 2 | 2 | 2 | 3 | 2 | 3 | 1 | 3 | 1 |

Table 1: Stress reduction scores for groups A, B, C in the treatment for stress reduction.

| Treatment group | Sample size | Mean | Standard deviation |
|---|---|---|---|
| Group A | 10 | 3.5 | 0.972 |
| Group B | 10 | 3.0 | 1.333 |
| Group C | 10 | 2.0 | 0.816 |
| Total | 30 | 2.83 | |

Table 2: Descriptive measures of groups A, B, C in the treatment for stress reduction dataset.

First, it was of interest to compare the mental treatment group (group A) and the physical training method group (group B).

**a)** Let $S_A$ and $S_B$ be the standard deviation from treatment group A and group B and let them be estimators of $\sigma_A$ and $\sigma_B$.

Test the hypothesis

$$H_0 : \sigma_A = \sigma_B \text{ vs. } H_1 : \sigma_A \neq \sigma_B$$

by calculating a 95% confidence interval for $\frac{\sigma_A}{\sigma_B}$.

Based on the data, can we conclude that the stress reduction scores are different for the two treatment groups (group A and group B)? Write down the null hypothesis and the alternative hypothesis and perform a two-sample t-test based on the summary statistics in Table 2. Use significance level $\alpha = 0.05$. What are the assumptions you need to make to use this test? What is the conclusion from the test?

**b)** A Wilcoxon rank-sum (Mann–Whitney) test was performed on the data for group A and B to test if the stress reduction scores are different for the two treatment groups (group A and group B).

Write down the null hypothesis and the alternative hypothesis for this test. From the MINITAB output found in Figure 1, the test statistics is $W_1 = 116$. Show how this value can be obtained from the data in Table 1. Explain briefly how the value of $W_1$ is used to test the hypothesis.

Use significance level $\alpha = 0.05$.

What are the assumptions you need to use this test?
Which of the tests in a) (second half) and b) would you consider most appropriate for the data? You may base your answer on Figure 2.

```
Mann-Whitney Test and CI

         N  Median
Group A  10   4,000
Group B  10   3,000

Point estimate for ETA1-ETA2 is 1
95,5 Percent CI for ETA1-ETA2 is (-1,000;1,999)
W = 116,0
Test of ETA1 = ETA2 vs ETA1 not = ETA2 is significant at 0,4274
The test is significant at 0,4073 (adjusted for ties)
```

Figure 1: Printout from Wilcoxon rank-sum test (Mann-Whitney) for the treatment for stress reduction dataset.

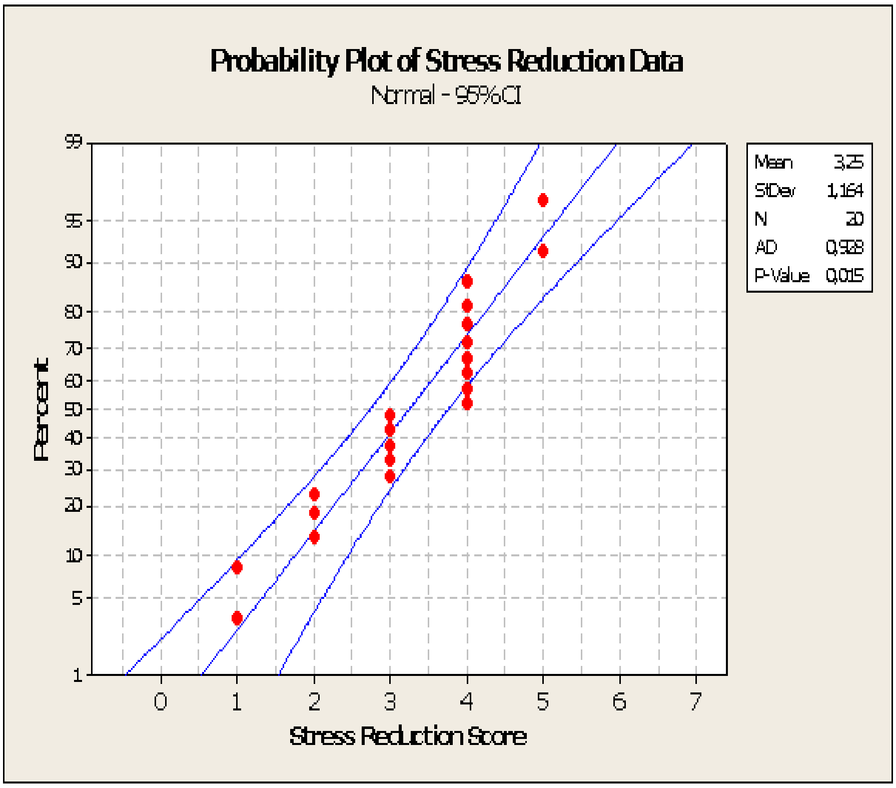



Figure 2: A normal plot based on the treatment for stress reduction data set, for merged data for group A and group B.

In the rest of the problem you shall use all three stress treatment groups, A, B and C. Boxplots for the three treatment groups are given in Figure 3.

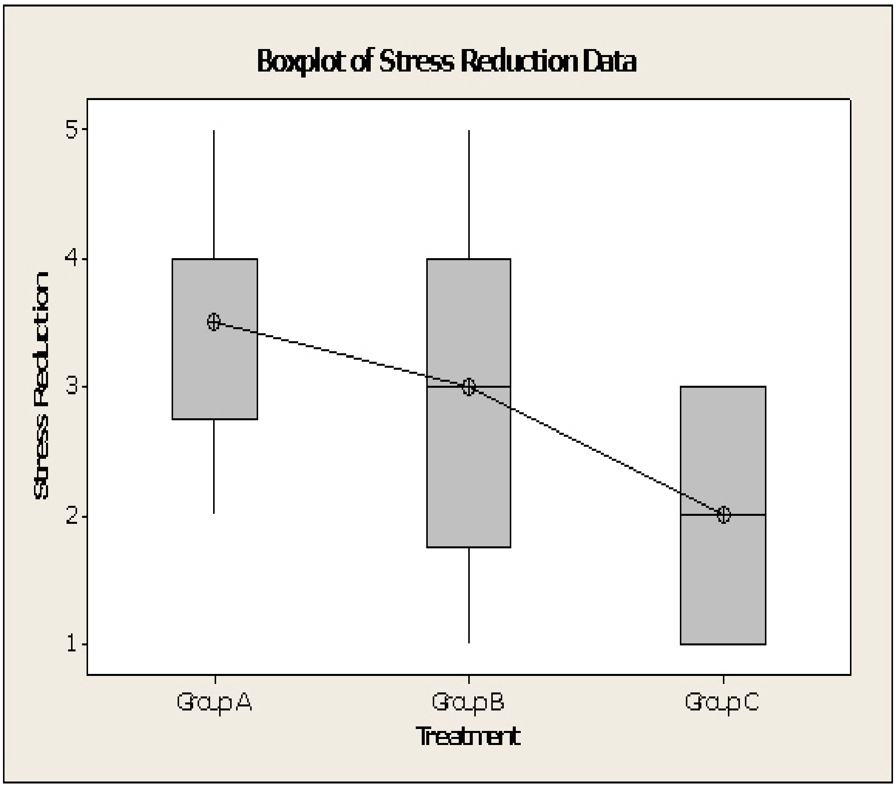



Figure 3: Boxplots for group A, B and C in the treatment for stress reduction data set.

**c)** It was of interest to find out whether there are differences between the three treatment groups regarding stress reduction effect.

First calculate the sums of squares, SSA and SSE, by using the summary statistics in Table 2.

Perform a one-way analysis of variance to test the null hypothesis

$$H_0 : \mu_A = \mu_B = \mu_C.$$

Use significance level $\alpha = 0.05$. What are the assumptions you need to make to use this test? What is the conclusion from the test?

**d)** From the result in c) it was decided to further investigate if any of the differences $\mu_A - \mu_B$, $\mu_A - \mu_C$ and $\mu_B - \mu_C$ are significantly different from 0.

Use the Bonferroni method for this problem. The family-wise error rate (FWER), i.e. the probability of making at least one type I error, should be controlled at level $\alpha$. Use $\alpha = 0.05$.

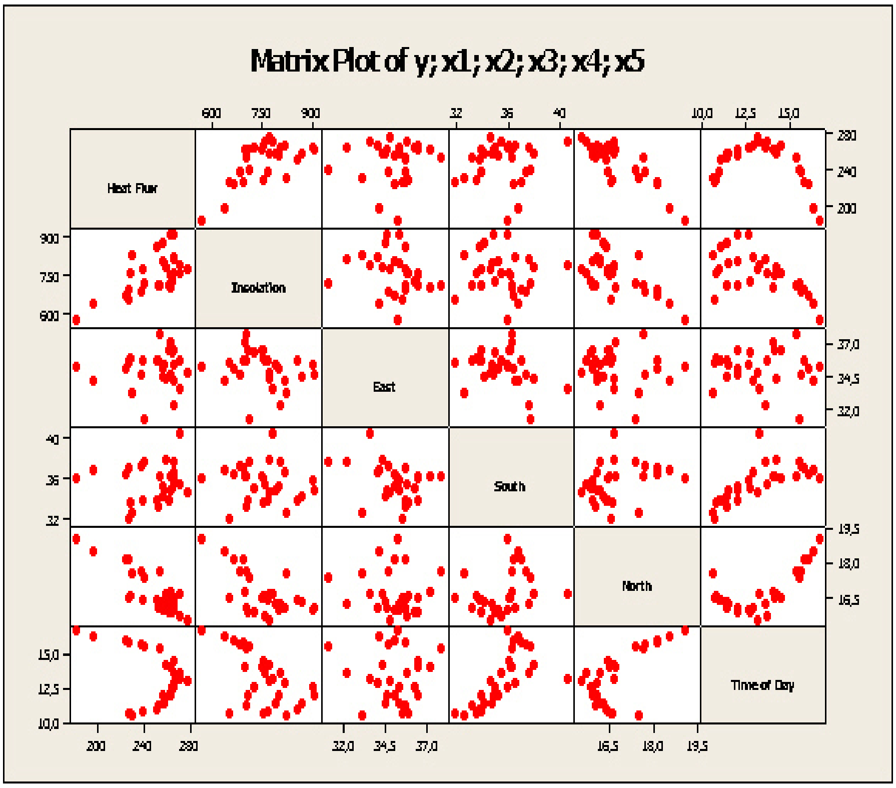
## Problem 2 Heat flux

Energy engineers are interested in thermal energy from the sun, as part of development of solar energy. Heat flux is the rate of heat energy transfer through a given surface per unit time. The heat flux is measured as part of a solar thermal energy test. An engineer is interested in determining how the total heat flux is predicted by the variables: insolation, the position of the east, south, and north focal points, and the time of day. $n = 29$ thermal energy tests were taken.

The following description is given.

- $y$: the total heat flux, in kilowatts
- $x_1$: the insolation value, in watts/m2
- $x_2$: the position of the focal point in the east direction, in inches
- $x_3$: the position of the focal point in the south direction, in inches
- $x_4$: the position of the focal point in the north direction, in inches
- $x_5$: the time of day that the data were collected

A pairwise scatter plot and a correlation matrix of the variables are found in Figure 4.




| | y | x1 | x2 | x3 | x4 |
|---|---|---|---|---|---|
| x1 | 0,628 | | | | |
| x2 | 0,102 | -0,204 | | | |
| x3 | 0,112 | -0,107 | -0,329 | | |
| x4 | -0,849 | -0,634 | -0,117 | 0,287 | |
| x5 | -0,351 | -0,584 | -0,065 | 0,697 | 0,685 |

Figure 4: Pairwise scatter plots (upper part) and pairwise Pearson correlation (lower part) between variables $y$, $x_1$, $x_2$, $x_3$, $x_4$ and $x_5$ in the heat flux data set.

A multiple linear regression was fitted to the data with $y$ as response and $x_1$, $x_2$, $x_3$, $x_4$ and $x_5$ as explanatory variables. Let ($y_i$, $x_{1i}$, $x_{2i}$, $x_{3i}$, $x_{4i}$ and $x_{5i}$) denote the measurements from the $i$th thermal energy test, where $i = 1, ..., 29$. Define the full model (model A):

$$\text{Model A} \quad y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \beta_4 x_{4i} + \beta_5 x_{5i} + \epsilon_i, \tag{1}$$

where $\epsilon_i$ are i.i.d. $N(0, \sigma^2)$ for $i = 1, ..., n$. The MINITAB output from a statistical analysis of model A is found in Figure 5. Plots of standardized residuals are found in Figure 6.

| Predictor | Coef | SE Coef | T | P |
|---|---|---|---|---|
| Constant | 325,44 | 96,13 | 3,39 | 0,003 |
| Insolation | 0,06753 | 0,02899 | 2,33 | 0,029 |
| East | 2,552 | 1,248 | 2,04 | 0,053 |
| South | 3,800 | 1,461 | 2,60 | 0,016 |
| North | -22,949 | 2,704 | -8,49 | 0,000 |
| Time of Day | 2,417 | 1,808 | 1,34 | ? |

```
S = ?   R-Sq = ?%   R-Sq(adj) = 87,7%

PRESS = 3109,95   R-Sq(pred) = 78,82%
```

Analysis of Variance

| Source | DF | SS | MS | F | P |
|---|---|---|---|---|---|
| Regression | 5 | ? | 2639,1 | 40,84 | 0,000 |
| Residual Error | 23 | ? | 64,6 | | |
| Total | 28 | 14681,9 | | | |

Figure 5: Printout from fitting linear regression model A for the heat flux dataset.

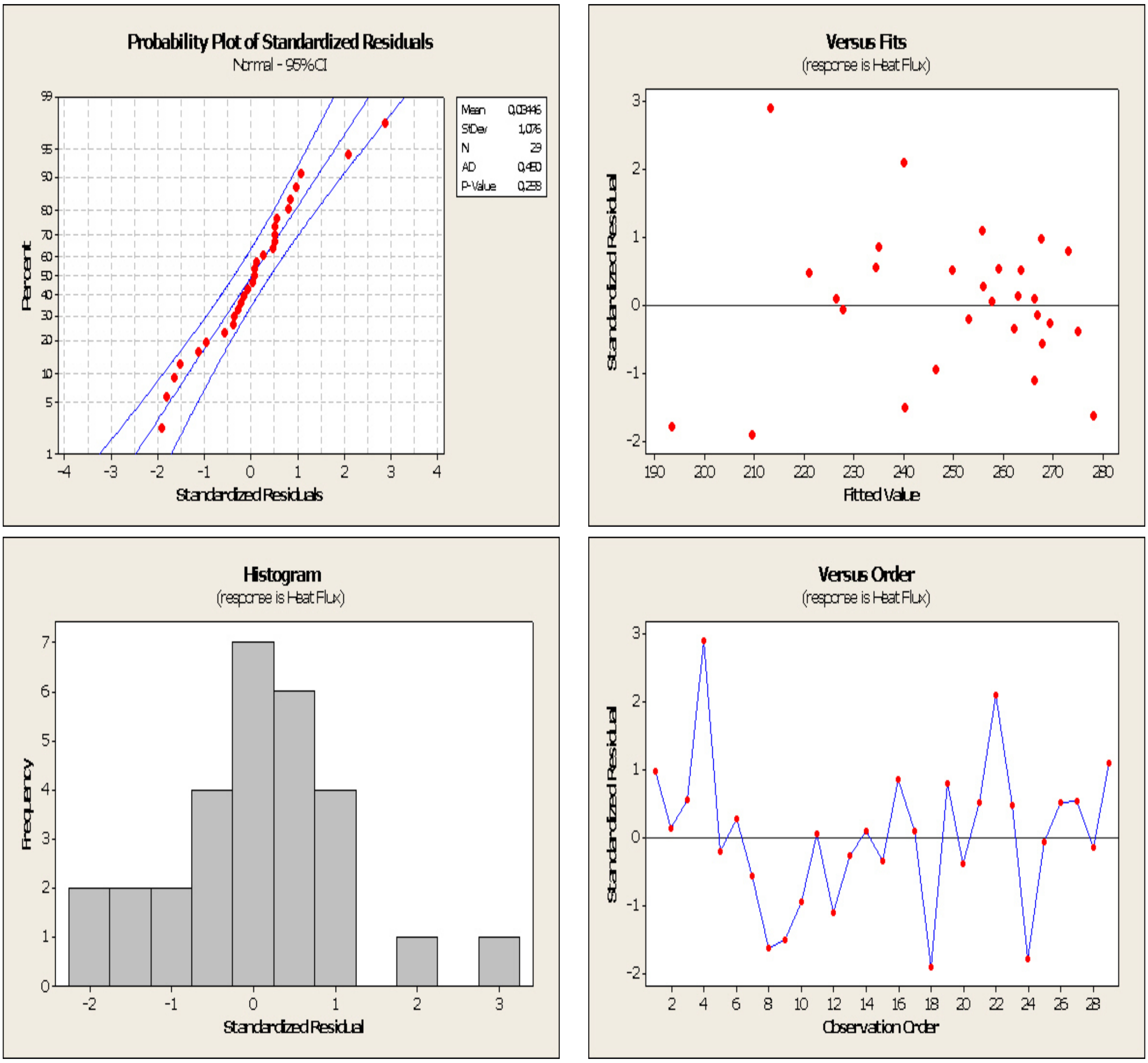



Figure 6: Residual plots (normal plot based on standardized residuals in the upper left panel, standardized residual versus fitted values in the upper right panel, histogram based on standardized residuals in lower left panel and standardized residual versus observation order in the lower right panel) for linear regression model A in the heat flux data set.

**a)** Write down the estimated regression equation.

Now turn to the estimated regression coefficient for $X_5$, ***Time of Day***, in this model. Is the effect of $X_5$, ***Time of Day***, significant in this model?

Calculate the $R^2$ and explain how you can interpret this value.

What is an appropriate estimate for $\sigma$? Give the numerical value of the estimate.

Is a significant amount of variation explained by the model? Write down the null hypothesis and the alternative hypothesis. Choose a test statistics and perform a hypothesis test. Use significance level $\alpha = 0.05$.

**b)** Which model assumptions were made in the linear regression model A in Eqn.1? Explain and define what a residual is.

Figure 6 shows some residual plots for model A. Do the residual plots indicate that these assumptions for linear regression are satisfied? Justify your answer by commenting briefly on the plots in Figure 6. How should they ideally look if the model was correct?

We are now interested in comparing different regression models where combinations of the covariates $x_1$, $x_2$, $x_3$, $x_4$ and $x_5$ are present, in a best subset regression. Assume that an intercept, $\beta_0$, is present in the regression model.

The MINITAB output from fitting different regression models to the data are presented in Figure 7. Each row in Figure 7 corresponds to one model. The number of explanatory variables included in each model (in addition to the intercept, $\beta_0$) is found in the column labeled $Vars$. The two best models of each number of explanatory variables is reported. The $X$'s indicate which variables that are found in the model.

| Vars | R-Sq | R-Sq(adj) | R-Sq(pred) | Mallows Cp | S | x 1 | x 2 | x 3 | x 4 | x 5 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 72,1 | 71,0 | 66.9 | 38,5 | 12,328 | | | | X | |
| 1 | 39,4 | 37,1 | 26.3 | 112,7 | 18,154 | X | | | | |
| 2 | 85,9 | 84,8 | 81.4 | 9,1 | 8,9321 | | | X | X | |
| 2 | 82,0 | 80,6 | 74.2 | 17,8 | 10,076 | | | | X | X |
| 3 | 87,4 | 85,9 | 79.0 | 7,6 | 8,5978 | | X | X | X | |
| 3 | 86,5 | 84,9 | 81.4 | 9,7 | 8,9110 | X | | X | X | |
| 4 | 89,1 | 87,3 | 80.6 | 5,8 | 8,1698 | X | X | X | X | |
| 4 | 88,0 | 86,0 | 79.3 | 8,2 | 8,5550 | X | | X | X | X |
| 5 | 89,9 | 87,7 | 78.8 | 6,0 | 8,0390 | X | X | X | X | X |

Figure 7: Printout from statistical analysis of the heat flux dataset.

**c)** Based on the result in Figure 7, which model would you choose as the "best" model for this dataset? Explain your choice.

To answer this question you should also explain how $R^2$, $R^2_{adj}$, $R^2_{pred}$, Mallows $C_p$ and $S$ are defined and how you can use them to compare the different regression models.

## Problem 3 Time to tie shoe laces

A pilot study was conducted where the time it takes, in seconds, for nine-year-old children to tie their shoe laces were measured.

The total number of children was $n = 250$. The time used given in intervals, and the number of children in each interval are found in the following table.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Time | (0,10] | (10,20] | (20,30] | (30,40] | (40,50] | (50,60] | $(60,\infty]$ |
| Frequency | 9 | 16 | 51 | 72 | 79 | 11 | 12 |
| Normal probability | 0.0062 | 0.0606 | 0.2417 | 0.3829 | 0.2417 | 0.0606 | 0.0062 |

**a)** For further studies on time for children to tie shoe laces the researchers were interested in testing if they can assume that the total time to tie shoe laces are normally distributed with mean 35 and standard deviation 10.

Calculate the probability, under the given assumption, that the total time to tie shoe laces for a random child, lies in the interval $(30, 40]$. Compare this result with the number found in the row labeled "Normal probability" and the column labeled "4".

Write down the null hypothesis and the alternative hypothesis and perform a hypothesis test to test if this data comes from a normal distribution with mean 35 and standard deviation 10. Base this test on the table above. Use a 5% level of significance.

What is the conclusion based on this test?

How could you instead test the null hypothesis that the total time to tie shoe laces is normally distributed, $N(\mu, \sigma)$, when $\mu$ and $\sigma$ are unspecified? You need only comment in words.

(Hint: If $X$ is binomial distributed, then $E(X) = n \cdot p$.)