Chapter 13

• One-way ANOVA. Assumptions: k samples of sizes $n_1, n_2, ..., n_k$ (from k populations) are independent and normally distributed with means $\mu_1, \mu_2, ..., \mu_k$ and common variance σ^2.

Hypothesis

$$H_0 : \mu_1 = \mu_2 = ... = \mu_k$$

$$H_1 : \mu_i \neq \mu_j \text{ for at least one pair } (i, j)$$

• Model. Data

$$Y_{ij} = \mu_i + \epsilon_{ij}, \ i = 1, ..., k, \ j = 1, ..., n_i, \ \epsilon_{ij} \sim N(0, \sigma^2).$$

An alternative form

$$Y_{ij} = \mu + \alpha_i + \epsilon_{ij}, \ \mu = \frac{1}{k}\sum_{i=1}^{k} \mu_i$$

(α_i is called the effect of the i-th treatment)

$$H_0 : \alpha_1 = \alpha_2 = ... = \alpha_k = 0$$

$$H_1 : \alpha_i \neq 0 \text{ for at least one } i$$

• Means and sums of squares:

$$\bar{Y}_{i.} = \frac{1}{n_i}\sum_{j=1}^{n_i} Y_{ij}, \ \bar{Y}_{..} = \frac{1}{N}\sum_{i=1}^{k} n_i \bar{Y}_{i.} \ \left(N = \sum_{i=1}^{k} n_i\right),$$

$$SST = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(Y_{ij} - \bar{Y}_{..})^2$$

(total sum of squares),

$$SSA = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(\bar{Y}_{i.} - \bar{Y}_{..})^2$$

(treatment sum of squares),

$$SSE = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(Y_{ij} - \bar{Y}_{i.})^2$$

(error sum of squares).

• Properties:
1) $SST = SSA + SSE$,
2) SSA and SSE are independent,
3) $SSE/\sigma^2 \sim \chi^2_{N-k}$,
4) $SSA/\sigma^2 \sim \chi^2_{k-1}$ under H_0.

• Test statistic

$$F = \frac{SSA/(k-1)}{SSE/(N-k)}.$$

Under H_0 F has the Fisher distribution with $k-1$ and $N-k$ degrees of freedom. Test: if $F \geq f_{\alpha,k-1,N-k}$, then H_0 is rejected.

- Bartlett's test (testing equality of the variances). Sample variances:

$$S_i^2 = \frac{1}{n_i - 1} \sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_{i.})^2$$

Pooled variance

$$S^2 = \frac{1}{N-k} \sum_{i=1}^{k} (n_i - 1) S_i^2.$$

Test statistic

$$B = \frac{\left[(S_1^2)^{n_1-1}(S_2^2)^{n_2-1} \cdots (S_k^2)^{n_k-1}\right]^{1/(N-k)}}{S^2}.$$

If $n_1 = n_2 = ... = n_k = n$, then H_0 is rejected if

$$B \leq b_k(\alpha; n)$$

where $b_k(\alpha; n)$ is taken from the table of the Bartlett distribution. If n_i are different, then the critical value is approximative (see the book).

- A contrast

$$w = \sum_{i=1}^{k} c_i \mu_i$$

where $\sum_{i=1}^{k} c_i = 0$. Many hypotheses about μ-s can be formulated as

$$H_0 : w = 0 \quad H_1 : w \neq 0.$$

Test: if

$$\frac{(\sum_{i=1}^{k} c_i \bar{Y}_i)^2}{S_p^2 \sum_{i=1}^{k} (c_i^2 / n_i)} \geq f_{\alpha,1,N-k},$$

then H_0 is rejected.