

Chapter 17

- Control chart categories: variables and attributes (proportion defective).
- Control of the mean. Let $\bar{X}_1, \bar{X}_2, \dots, \bar{X}_k$ be averages in subgroups. The overall average

$$\bar{\bar{X}} = \frac{1}{k} \sum_{i=1}^k \bar{X}_i$$

is the centerline of the control chart. The upper control limit and the lower control limit are

$$\text{UCL} = \bar{\bar{X}} + A_2 \bar{R}, \quad \text{LCL} = \bar{\bar{X}} - A_2 \bar{R}$$

where $\bar{R} = \frac{1}{k} \sum_{i=1}^k R_i$, $R_i = X_{\max,i} - X_{\min,i}$,

$$A_2 = \frac{3}{d_2 \sqrt{n}}.$$

Values of d_2 and A_2 are given for various sample sizes in a table.

- Control of the variability. R -chart (plot of the sample range): \bar{R} is the centerline, $\text{UCL} = \bar{R} D_4$, $\text{LCL} = \bar{R} D_3$, where

$$D_4 = 1 + 3 \frac{d_3}{d_2}, \quad D_3 = 1 - 3 \frac{d_3}{d_2}.$$

Values of d_2 , d_3 , D_3 , D_4 are tabulated for various sample sizes.

- If n is large ($n > 10$), then

$$S = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2}$$

should be used instead of the range R (for both the mean and the variability).

To control the variability, \bar{S} is the centerline, $\text{LCL} = B_3 \bar{S}$, $\text{UCL} = B_4 \bar{S}$, where

$$B_3 = 1 - \frac{3}{c_4} \sqrt{1 - c_4^2}, \quad B_4 = 1 + \frac{3}{c_4} \sqrt{1 - c_4^2},$$

and c_4 is such that

$$E\left(\frac{\bar{S}}{c_4}\right) = \sigma.$$

To control the mean, $\bar{\bar{X}}$ is the centerline, $\text{LCL} = \bar{\bar{X}} - A_3 \bar{S}$, $\text{UCL} = \bar{\bar{X}} + A_3 \bar{S}$, where

$$A_3 = \frac{3}{c_4 \sqrt{n}}.$$

Values of c_4 , B_3 , B_4 , A_3 are tabulated for various sample sizes.

- Control charts for attributes (p -charts): centerline \bar{p} ,

$$\text{LCL} = \bar{p} - 3 \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}, \quad \text{UCL} = \bar{p} + 3 \sqrt{\frac{\bar{p}(1-\bar{p})}{n}},$$

where

$$\bar{p} = \frac{1}{m} \sum_{i=1}^m \hat{p}_i,$$

and \hat{p}_i is the fraction of defectives in the i -th sample.