Chapter 8

• Two independent samples of sizes n_1 and n_2 respectively. The first one is from a population with mean μ_1 and variance σ_1^2, the second one is from a population with mean μ_2 and variance σ_2^2. If $\bar{X}_1$ and $\bar{X}_2$ are sample means of the samples, then distribution of the statistic

$$\frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$$

is approximately standard normal.

• If (and only if!) X-s have a normal distribution, statistics $\bar{X}$ and S^2 are independent.

• Distribution of S^2. If $X_i \sim N(\mu, \sigma^2)$, then

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}.$$

• Student t-distribution with m degrees of freedom: distribution of

$$T = \frac{Z}{\sqrt{V/m}},$$

where Z and V are independent, $Z \sim N(0,1)$, $V \sim \chi^2_m$.

• If $X_i \sim N(\mu, \sigma^2)$, then

$$T = \sqrt{n}\frac{\bar{X} - \mu}{S}$$

has t-distribution with $n-1$ degrees of freedom.

• Fisher F-distribution with m and n degrees of freedom: distribution of

$$F = \frac{U/m}{V/n},$$

where U and V are independent, $U \sim \chi^2_m$, $V \sim \chi^2_n$.

• Two independent samples of sizes n_1 and n_2 respectively. The first one is from a normal population with variance σ_1^2, the second one is from a normal population with variance σ_2^2. If S_1^2 and S_2^2 are sample variances of the samples, then the statistic

$$\frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2}$$

has F-distribution with $n_1 - 1$ and $n_2 - 1$ degrees of freedom.