Solutions. TMA4255, 2018 (June)

1.
a) Under H_0 the test statistic of the test

$$\frac{\bar{X}-\bar{Y}}{\sqrt{\sigma_X^2/n+\sigma_Y^2/m}}$$

has the standard normal distribution, therefore the $P-$value is

$$p(x,y) = P_{\mu_X=\mu_Y}\left(\left|\frac{\bar{X}-\bar{Y}}{\sqrt{\sigma_X^2/n+\sigma_Y^2/m}}\right| \geq \frac{\bar{x}-\bar{y}}{\sqrt{\sigma_X^2/n+\sigma_Y^2/m}}\right) =$$

$$= 2\Phi(-1) \approx 0.32.$$

2.
a) We have to test

$$H_0 : \beta_1 = 0 \quad H_1 : \beta_1 \neq 0.$$

The test statistic is

$$T = \frac{\hat{\beta}_1}{\sqrt{S^2/\sum_{i=1}^{10}(x_i-\bar{x})^2}}$$

which (under H_0) has t-distribution with $10-2=8$ degrees of freedom. H_0 is rejected if $|T| \geq t_{0.025,8}$. In our case

$$T = \frac{1.112}{\sqrt{2.3/4.1}} = 1.49.$$

$t_{0.025,8} = 2.306$. H_0 is not rejected.

3.
a) Test statistics are

$$T_r = \frac{\hat{\beta}_r}{\mathrm{SD}(\hat{\beta}_r)}, \ r = 1, 2.$$

Under H_0 both test statistics have $t-$distribution with $24-3=21$ degrees of freedom. The null hypothesis is rejected if $|T_r| > t_{0.005,21}$. From the MINITAB output we obtain

$$T_1 = \frac{-10.025}{0.9466} = -10.61, \ T_2 = \frac{0.80842}{0.01653} = 48.91.$$

Since $t_{0.005,21} = 2.831$, both null hypotheses are rejected.

4.

a) The randomized block design should be used. The model is

$$Y_{ij} = \mu_j + \beta_i + \epsilon_{ij}, \;\; \epsilon_{ij} \sim N(0, \sigma^2),$$

$$i = 1, 2, ..., b \;(b = 6), \;\; j = 1, 2, ..., k \;(k = 5),$$

where μ_j is the treatment effect of instrument number j, β_i is the block effect of object number i, ϵ_{ij} are random errors.

We have $k-1 = 4$, $b-1 = 5$, $(b-1)(k-1) = 20$, $bk-1 = 29$. These are elements of the "DF"- column. The first element of the "F"- column is such a value f that $P(F_{4,20} > f) = 0.025$, where $F_{m,n}$ is a random variable having the $F-$distribution with m and n degrees of freedom. We find from the table of quantiles of $F-$distributions that it is 3.51. The second element of the "F"- column is such a value f that $P(F_{5,20} > f) = 0.05$. It is 2.71. Since MS = SS/DF, F = MSTR/MSE (for treatments) F = MSB/MSE (for blocks), and SSTOT = SSTR + SSB + SSE, we finally obtain

Source	DF	SS	MS	F	P
Instrument	4	8,0	2,00	3,51	0,025
Object	5	7,7	1,54	2,71	0,050
Error	20	11,4	0,57		
Total	29	27,1			

b) H_0 for the $P-$value in the first line is

$$H_0 : \;\mu_1 = \mu_2 = ... = \mu_5.$$

H_0 for the $P-$value in the second line is

$$H_0 : \;\beta_1 = \beta_2 = ... = \beta_6.$$

The institute is interested in the first $P-$value (differences in instruments). Since the $P-$value is smaller than the significance level, the null hypothesis is rejected.

5.

a) Let X be the number of observations greater than 20.0. The large-sample sign test: H_0 is rejected if

$$Z = \frac{X - n/2}{\sqrt{n/4}} \geq z_\alpha.$$

In our case ($X = 11$, $n = 15$)

$$Z = 1.8 > 1.645 = z_\alpha$$

therefore H_0 is rejected.

b) The large-sample Wilcoxon signed rank test: H_0 is rejected if

$$z = \frac{w - n(n+1)/4}{\sqrt{n(n+1)(2n+1)/24}} \geq z_\alpha,$$

where $w = \sum_{i=1}^{n} r_i z_i$, r_i – rank of $|y_i - 20|$, $z_i = 1$ if $y_i > 20$ and 0 otherwise.

i	1	2	3	4	5	6	7	8
$\lvert y_i - 20 \rvert$	0.8	7.6	5.6	12.2	2.3	0.5	3.9	0.2
r_i	3	12	10	15	5	2	7	1
z_i	0	1	1	1	0	1	1	1
i	9	10	11	12	13	14	15	
$\lvert y_i - 20 \rvert$	4.2	6.1	12.0	4.8	8.9	3.8	1.3	
r_i	8	11	14	9	13	6	4	
z_i	1	1	1	1	1	0	0	

In our case

$$w = 102$$

and

$$z = 2.39 > 1.645 = z_\alpha.$$

Thus H_0 is rejected.

c) Let

$$T = \sqrt{n}\frac{\bar{y} - \mu_0}{s}$$

where

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2.$$

The t-test for testing $H_0 : \mu = \mu_0$ versus $H_1 : \mu > \mu_0$: H_0 is rejected if $T \geq t_{\alpha,n-1}$. In our case $\mu_0 = 20$, $\alpha = 0.05$, $n = 15$, $t_{\alpha,n-1} = 1.761$, $T = 2.97$.

H_0 is rejected.