Chapter 9

• Two samples (of sizes n_1 and n_2 which are large enough for the normal approximation) with expectations μ_1, μ_2 and variances σ_1^2, σ_2^2. $(1-\alpha)$ confidence interval for $\mu_1 - \mu_2$.

If σ_1^2, σ_2^2 are known, then

$$\left[\bar{X} - \bar{Y} - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}, \bar{X} - \bar{Y} + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right].$$

If σ_1^2, σ_2^2 are unknown but equal $\sigma_1^2 = \sigma_2^2 = \sigma^2$, then

$$\left[\bar{X} - \bar{Y} - t_{\frac{\alpha}{2}, n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}, \bar{X} - \bar{Y} + t_{\frac{\alpha}{2}, n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right]$$

(S_p^2 is the pooled empirical variance).

If σ_1^2, σ_2^2 are unknown and unequal, then

$$\left[\bar{X} - \bar{Y} - t_{\frac{\alpha}{2}, m}\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}, \bar{X} - \bar{Y} + t_{\frac{\alpha}{2}, m}\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}\right],$$

where m is the nearest integer to

$$\frac{(S_1^2/n_1 + S_2^2/n_2)^2}{(S_1^2/n_1)^2/(n_1-1) + S_2^2/n_2)^2/(n_2-1)}.$$

• Paired data.

$$X_1, X_2, ..., X_n, \; EX_i = \mu_X \; (X_i = \mu_X + \epsilon_{1i}),$$

$$Y_1, Y_2, ..., Y_n, \; EY_i = \mu_Y \; (Y_i = \mu_Y + \epsilon_{2i}).$$

X-s are independent, Y-s are independent but X_i og Y_i are dependent for each i. $(1-\alpha)$ confidence interval for $\mu_X - \mu_Y$

$$\left[\bar{D} - t_{\alpha/2}\frac{S_D}{\sqrt{n}}, \bar{D} + t_{\alpha/2}\frac{S_D}{\sqrt{n}}\right]$$

where

$$D_i = X_i - Y_i, \; \bar{D} = \frac{1}{n}\sum_{i=1}^{n} D_i, \; S_D^2 = \frac{1}{n-1}\sum_{i=1}^{n}(D_i - \bar{D})^2.$$

Thus from two samples to one sample.