Chapter 10

• One sample $X_1, X_2, ..., X_n$, $EX_i = \mu$, $\text{Var}(X_i) = \sigma^2$. Testing $H_0 : \mu = \mu_0$. Alternatives a) $H_1 : \mu > \mu_0$, b) $H_1 : \mu < \mu_0$, c) $H_1 : \mu \neq \mu_0$. The level of significance α.

1) X-s are normally distributed (or $n > 30$), σ^2 is known. The test statistic

$$Z = \sqrt{n}\frac{\bar{X} - \mu_0}{\sigma}.$$

Z has the standard normal distribution under H_0 (exactly or approximately). The critical region: a) $Z \geq z_\alpha$, b) $Z \leq -z_\alpha$, c) $|Z| \geq z_{\alpha/2}$.

2) X-s are normally distributed (or $n > 30$), σ^2 is unknown. The test statistic

$$T = \sqrt{n}\frac{\bar{X} - \mu_0}{S}.$$

T has the t-distribution with $n-1$ degrees of freedom under H_0 (exactly or approximately). The critical region: a) $T \geq t_{\alpha,n-1}$, b) $T \leq -t_{\alpha,n-1}$, c) $|T| \geq t_{\alpha/2,n-1}$.

• Two samples $X_1, ..., X_n$ and $Y_1, ..., Y_m$ (from normal populations or n and m are large enough); $EX_i = \mu_X$, $\text{Var}(X_i) = \sigma_X^2$, $EY_i = \mu_Y$, $\text{Var}(Y_i) = \sigma_Y^2$. Testing $H_0 : \mu_X - \mu_Y = d_0$ (d_0 is a given number, usually $d_0 = 0$). Alternatives a) $H_1 : \mu_X - \mu_Y > d_0$, b) $H_1 : \mu_X - \mu_Y < d_0$, c) $H_1 : \mu_X - \mu_Y \neq d_0$. The level of significance α.

1) σ_X^2 and σ_Y^2 are known. The test statistic

$$Z = \frac{(\bar{X} - \bar{Y}) - d_0}{\sqrt{\sigma_X^2/n + \sigma_Y^2/m}}.$$

Z has the standard normal distribution under H_0 (exactly or approximately). The critical region: a) $Z \geq z_\alpha$, b) $Z \leq -z_\alpha$, c) $|Z| \geq z_{\alpha/2}$.

2) σ_X^2 and σ_Y^2 are unknown but equal: $\sigma_X^2 = \sigma_Y^2 = \sigma^2$. The test statistic

$$T = \frac{(\bar{X} - \bar{Y}) - d_0}{S_p\sqrt{1/n + 1/m}}$$

where

$$S_p^2 = \frac{(n-1)S_X^2 + (m-1)S_Y^2}{n+m-2}.$$

T has the t-distribution with $n + m - 2$ degrees of freedom under H_0 (exactly or approximately). The critical region: a) $T \geq t_{\alpha,n+m-2}$, b) $T \leq -t_{\alpha,n+m-2}$, c) $|T| \geq t_{\alpha/2,n+m-2}$.

• Test concerning variance (one sample).

Testing $H_0 : \sigma^2 = \sigma_0^2$. Alternatives a) $H_1 : \sigma^2 > \sigma_0^2$, b) $H_1 : \sigma^2 < \sigma_0^2$, c) $H_1 : \sigma^2 \neq \sigma_0^2$. Signifikansnivå α.

The test statistic

$$\chi^2 = \frac{(n-1)S^2}{\sigma_0^2}.$$

χ^2 has chi-squared distribution with $n-1$ degrees of freedom under H_0. Critical region: a) $\chi^2 \geq \chi^2_{\alpha,n-1}$, b) $\chi^2 \leq \chi^2_{1-\alpha,n-1}$, c) $\chi^2 \leq \chi^2_{1-\alpha/2,n-1}$ or $\chi^2 \geq \chi^2_{\alpha/2,n-1}$.

• Two samples $X_1, ..., X_n$ and $Y_1, ..., Y_m$ (from normal populations or n and m are large enough); $\mathrm{Var}(X_i) = \sigma^2_X$, $\mathrm{Var}(Y_i) = \sigma^2_Y$. Testing $H_0 : \sigma^2_X = \sigma^2_Y$. Alternatives a) $H_1 : \sigma^2_X > \sigma^2_Y$, b) $H_1 : \sigma^2_X < \sigma^2_Y$, c) $H_1 : \sigma^2_X \neq \sigma^2_Y$. The level of significance α.

The test statistic

$$F = \frac{S^2_X}{S^2_Y}.$$

Critical region: a) $F \geq f_{\alpha,n-1,m-1}$, b) $F \leq f_{1-\alpha,n-1,m-1}$, a) $F \geq f_{\alpha/2,n-1,m-1}$ or $F \leq f_{1-\alpha/2,n-1,m-1}$