Chapter 16

• Median $\tilde{\mu}$ of a random variable X is defined as such a value that

$$P(X \geq \tilde{\mu}) \geq \frac{1}{2}, \quad P(X \geq \tilde{\mu}) \leq \frac{1}{2}.$$

If X is continuous, then

$$P(X \geq \tilde{\mu}) = P(X > \tilde{\mu}) = P(X < \tilde{\mu}) = P(X \leq \tilde{\mu}) = \frac{1}{2}.$$

• Sign test. A sample $Y_1, Y_2, ..., Y_n$ – independent continuous random variables with median $\tilde{\mu}$. The hypothesis $H_0 : \tilde{\mu} = \tilde{\mu}_0$ is tested. Alternatives: a) $H_1 : \tilde{\mu} > \tilde{\mu}_0$, b) $H_1 : \tilde{\mu} < \tilde{\mu}_0$, c) $H_1 : \tilde{\mu} \neq \tilde{\mu}_0$. The significance level α. Let X be the number of Y-s which are greater than $\tilde{\mu}_0$. Test statistic

$$Z = \frac{X - n/2}{\sqrt{n/4}}.$$

$Z \sim N(0, 1)$ (approximately) under H_0. Critical regions: a) $Z \geq z_\alpha$, b) $Z \leq -z_\alpha$, c) $|Z| \geq z_{\alpha/2}$.

• Wilcoxon signed-rank test. A sample $Y_1, Y_2, ..., Y_n$ – independent identically distributed continuous symmetric (about expectation μ) random variables. The hypothesis $H_0 : \mu = \mu_0$ is tested. Alternatives: a) $H_1 : \mu > \mu_0$, b) $H_1 : \mu < \mu_0$, c) $H_1 : \mu \neq \mu_0$. The significance level α.

Rank

$$|Y_1 - \mu_0|, |Y_2 - \mu_0|, ..., |Y_n - \mu_0|.$$

Let R_i be the rank of $|Y_i - \mu_0|$. Define $X_i = 1$ if $Y_i > \mu_0$ and $X_i = 0$ if $Y_i < \mu_0$. Denote

$$W = \sum_{i=1}^{n} R_i X_i.$$

Test statistic is

$$Z = \frac{W - n(n+1)/4}{\sqrt{n(n+1)(2n+1)/24}}.$$

$Z \sim N(0, 1)$ (approximately) under H_0. Critical regions: a) $Z \geq z_\alpha$, b) $Z \leq -z_\alpha$, c) $|Z| \geq z_{\alpha/2}$.