Chapter 16

• Wilcoxon signed-rank test. A sample $Y_1, Y_2, ..., Y_n$ – independent identically distributed continuous symmetric (about expectation μ) random variables. The hypothesis $H_0 : \mu = \mu_0$ is tested. Alternatives: a) $H_1 : \mu > \mu_0$, b) $H_1 : \mu < \mu_0$, c) $H_1 : \mu \neq \mu_0$. The significance level α.

Rank

$$|Y_1 - \mu_0|, |Y_2 - \mu_0|, ..., |Y_n - \mu_0|.$$

Let R_i be the rank of $|Y_i - \mu_0|$. Define $X_i = 1$ if $Y_i > \mu_0$ and $X_i = 0$ if $Y_i < \mu_0$. Denote

$$W = \sum_{i=1}^{n} R_i X_i.$$

Test statistic is

$$Z = \frac{W - n(n+1)/4}{\sqrt{n(n+1)(2n+1)/24}}.$$

$Z \sim N(0, 1)$ (approximately) under H_0. Critical regions: a) $Z \geq z_\alpha$, b) $Z \leq -z_\alpha$, c) $|Z| \geq z_{\alpha/2}$.

• Wilcoxon rank-sum test (Mann-Whitney test). Observations

$$X_1, X_2, ..., X_{n_1}, Y_{n_1+1}, Y_{n_1+2}, ..., Y_{n_1+n_2}.$$

X-s are iid with median $\tilde{\mu}_1$, Y-s are iid with median $\tilde{\mu}_2$, X-s and Y-s are independent. The hypothesis $H_0 : \tilde{\mu}_1 = \tilde{\mu}_2$ is tested. Alternatives: a) $H_1 : \tilde{\mu}_1 > \tilde{\mu}_2$, b) $H_1 : \tilde{\mu}_1 < \tilde{\mu}_2$, c) $H_1 : \tilde{\mu}_1 \neq \tilde{\mu}_2$. The significance level α. Rank all

$$X_1, X_2, ..., X_{n_1}, Y_{n_1+1}, Y_{n_1+2}, ..., Y_{n_1+n_2}.$$

Denote their ranks by $R_1, R_2, ..., R_{n_1+n_2}$. Let

$$W = \sum_{i=1}^{n_1} R_i.$$

Test statistic is

$$Z = \frac{W - n_1(n_1 + n_2 + 1)/2}{\sqrt{n_1 n_2 (n_1 + n_2 + 1)(2n+1)/12}}.$$

$Z \sim N(0, 1)$ (approximately) under H_0. Critical regions: a) $Z \geq z_\alpha$, b) $Z \leq -z_\alpha$, c) $|Z| \geq z_{\alpha/2}$.

• Kruskal-Wallis test. Nonparametric analog of the ANOVA F-test. k samples of sizes $n_1, n_2, ..., n_k$ are independent with means $\mu_1, \mu_2, ..., \mu_k$. Hypothesis

$$H_0 : \mu_1 = \mu_2 = ... = \mu_k$$

$$H_1 : \mu_i \neq \mu_j \text{ for at least one pair } (i, j)$$

Observations are replaced by ranks. Let $R_1, R_2, ..., R_k$ be sums of ranks in the samples. Test statistic is (let $n = n_1 + n_2 + ... + n_k$)

$$H = \frac{12}{n(n+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 3(n+1).$$

$H \sim \chi^2_{k-1}$ under H_0. If $H \geq \chi^2_{\alpha,k-1}$, then H_0 is rejected.