Repetition week 12.

Two-way analysis of variance

Model

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \ \alpha\beta \ _{ij} + \varepsilon_{ijk} \begin{cases} \quad\quad \mathbf{N}\ \ 0,\sigma^2 \quad \text{and independent} \\ i = 1,2,\cdots,a, \ \ j = 1,2,\cdots,b, \ k = 1,2,\cdots,n \end{cases}$$

1. $H_0 : \alpha_1 = \alpha_2 = \cdots = \alpha_a = 0 \quad H_1$: at least one is different from 0.
2. $H_0 : \beta_1 = \beta_2 = \cdots = \beta_b = 0 \quad H_1$: at least one is different from 0.
3. $H_0 : \alpha\beta_{11} = \alpha\beta_{12} = \cdots = \alpha\beta_{ab} = 0 \quad H_1$: at least one is different from 0.

Sources	SS	DF	MS	F
A	$SS_A = nb\sum_{i=1}^{a} \ y_{i..} - y_{...} \ ^2$	$a-1$	$\frac{SS_A}{a-1}$	$F = \frac{MS_A}{MS_E}$
B	$SS_B = na\sum_{j=1}^{b} \ y_{.j.} - \bar{y}_{...} \ ^2$	$b-1$	$\frac{SS_B}{b-1}$	$F = \frac{MS_B}{MS_E}$
Interaction AB	$SS_{AB} = n\sum_{i=1}^{a}\sum_{j=1}^{b} \ y_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...} \ ^2$	$a-1 \ \ b-1$	$\frac{SS_{AB}}{a-1 \ \ b-1}$	$F = \frac{MS_{AB}}{MS_E}$
Error	$SS_E = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} \ y_{ijk} - \bar{y}_{ij.} \ ^2$	$ab \ \ n-1$	$\frac{SS_E}{ab \ \ n-1}$	
Total	$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} \ y_{ijk} - \bar{y}_{...} \ ^2$	$abn-1$		

Control Charts

Let T be a test statistics based on a sample $X_1, X_2, \ldots, X_n$.

Control Chart: LCL = $\mu_T - k\sigma_T$, CL = μ_T, UCL = $\mu_T + k\sigma_T$

Reduce the probability of false alarm: increase k.

Increase the probability of detecting out of control: increase n.

$\bar{X}bar - R\ Chart$

k samples: Test statistics: $\bar{X}_1, \ldots, \bar{X}_k,\ R_1, \ldots, R_k$.

$$\bar{\bar{X}} = \frac{\sum_{i=1}^{k} \bar{X}_i}{k}, \qquad \bar{R} = \frac{\sum_{i=1}^{k} R_i}{k}.$$

$\bar{X}bar$: LCL= $\bar{\bar{X}} + \frac{3\bar{R}}{d_2\sqrt{n}}$, CL = $\bar{\bar{X}}$, UCL= $\bar{\bar{X}} + \frac{3\bar{R}}{d_2\sqrt{n}}$.

$R\ Chart$: LCL = $\bar{R} - \frac{3\bar{R}d_3}{d_2}$, CL = $\bar{R}$, UCL = $\bar{R} + \frac{3\bar{R}d_3}{d_2}$