TMA4265 Stochastic Processes
Week 42-43 – Exercises

English version of Problem 2 of Mai 2003 exam

A Poisson process $\{N(t); t \geq 0\}$ with rate λ has (as known) the following properties

$$\text{i) } P\{N(h) = 1\} = \lambda h + o(h) \quad \text{and} \quad \text{ii) } P\{N(h) \geq 2\} = o(h).$$

Let $\{N_A(t), t \geq 0\}$ and $\{N_B(t), t \geq 0\}$ be two independent Poisson processes with rates λ_A and λ_B, respectively. Let further $N(t) = N_A(t) = N_B(t)$.

a) Show with precise computations, i.e. using the o(h)-notation, that $N(t)$ fullfils the properties i) and ii) setting $\lambda = \lambda_A + \lambda_B$.

Use known properties from the Poisson process to answer the following questions where $\{N_A(t), t \geq 0\}$, $\{N_B(t), t \geq 0\}$ and $\{N(t); t \geq 0\}$ are specified as above and $\lambda_A = 1$ and $\lambda_B = 2$.

b) Derive the probabilities

$$P\{N(1) \leq 2\} \quad , \quad P\{N_A(1) = 1 | N(1) = 1\} \quad \text{and} \quad P\{N(1) \leq 2 | N_A(0.5) = 1\}.,$$