Part 4: DoE

Performing a full 2^k factorial expr.

Two important aspects:

a) The run order is random, so that potential external factors are not confused/confounded with experimental factors.

b) Each experiment is a genuine run replicate, that is, reflects the total variability of the experiment.

Blocking

We will perform a 2^3 experiment, but have to use two batches of raw material $\Rightarrow$ need to divide the 8 runs into two groups. What is the best way to do this?

Solution: use the ABC column to define the blocks, ABC is the block generator.

ABC = -1 : use batch 1
ABC = +1 : 2

The block "variable" will be a new regressor replacing the ABC factor.

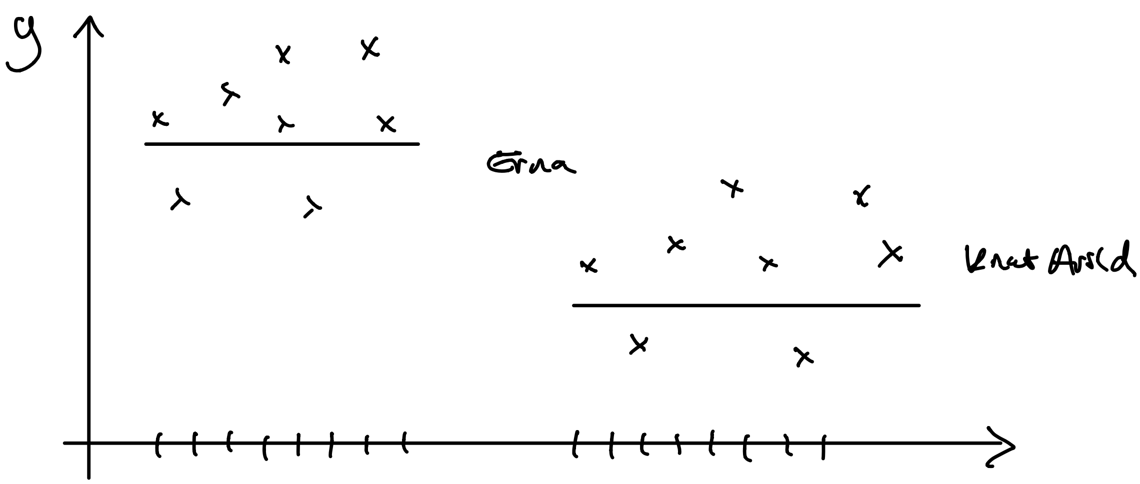
What would happen if I did not include the block as a regressor/covariate in the analysis?

$\Rightarrow$ SSE will be big.

Example: person as block

Erna and Knut Arild want to perform an 2^3 experiment together, and to get 16 obs. they will both conduct the same physical 2^3 experiments.

Should then a covariate telling who did each experiment be added to the regression model?



If the above figure gives a correct picture of the experiment NOT including a person covariate will make SSE very large, and will therefore give only non significant effects.

2^3 in four blocks

To divide the $2^3 = 8$ runs into 4 blocks we need two block generators. The best solution is to use AB and AC as generators for the blocks:

Block	AB	AC	
1	−	−	⇐ (run 2 and 7 in std. order)
2	−	+	
3	+	−	
4	+	+	

Then the block effect will not be confounded with the main effect A, B, C or ABC interaction.
But will be confounded with AB, AC and also

$$AB \cdot AC = \underbrace{A^2}_{\substack{\text{col. with } +1 \\ = \\ I}} BC = BC$$

Q: What if ABC and BC were to be chosen as block generators?

$ABC \cdot BC = AB^2C^2 = A$ ⇐ blocks will be confounded with the A effect

Fractional factorial designs

<u>Observation</u>: when the number of factors (k) is large, it may not be optimal to perform a full 2^k factorial design, due to the possible redundancy of the design.

higher order interactions tend to be smaller than lower order interactions

<u>Solution:</u> only perform a fraction of the full design!

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

Now: move in the opposite direction to solve this.

We have a full 2^3 factorial design with factors A, B, C, but we also want to have a new factor D in the experiment.

Possible solution: let the ABC column define the levels of factor D.

1) $D = ABC$ is called the "generator" of the design.

2) $I = D \cdot D = D \cdot ABC = ABCD$ is called the "defining relation" of the design.

column of 1s

3) The number of letters (lenth) of the (shortest) defining relation is called the "resolution of the design", and is denoted by Roman numerals. Here: $\underline{IV}$

4) Finaly: this is called a half-fraction of a 2^4 design

NOTATION

$2^{4-1}_{\underline{IV}}$ design with $I = ABCD$ as defining relation and $D = ABC$ as generator.

We may perform 8 experiments and estimate 8 parameter

may use Lenth's method to assess significance

with 4 factors there are:

$4 = \binom{4}{1}$ main effect A, B, C, D

$6 = \binom{4}{2}$ two-way interactions $AB, AC, \ldots, CD$

$4 = \binom{4}{3}$ three-way interaction ABC, ACD, BCD, ABD

$1 = \binom{4}{1}$ four-way interaction $ABCD$

$4+6+4+1 = 15$ possible effect (+1 intercept)

Q: what can we estimate?

The "alias-structure" defines which effects are confounded.

Obvious: Since $D = ABC$, then D and ABC are confounded.
$\hat{D}$ may actually be $\hat{D} + \widehat{ABC}$

Method: We want to find if any effects are confounded with A. We multiply A with the defining relation, $I = ABCD$.

1) Main effects:

$$A = A \cdot I = A \cdot ABCD = \overset{\overset{1s}{\parallel}}{A^2}\, BCD = BCD$$

that is A and BCD column are equal

$$B = B \cdot I = B \cdot ABCD = ACD$$
$$C = C \cdot I = C \cdot ABCD = ABD$$
$$D = D \cdot I = D \cdot ABCD = ABC$$

All main effects are confounded with 3-way interactions

$$l_A = A + BCD$$

we think that we estimate A, but we actually estimate $A + BCD$, BUT if 3-way interactions are small ⇒ OK!

2) 2-way:

$AB = AB \cdot I = AB \cdot ABCD = \underline{CD}$ $\qquad l_{AB} = AB + CD$

$AC =$ $\qquad l_{AC} = AC + BD$

$AD =$ $\qquad l_{AD} = AD + BC$

3) 3-way: already done $\leftarrow$ main effects

4) $I = ABCD$ is confounded with intercept.