

Fractional factorial designs (cont.)

not all possible experiments are performed - but a fraction
 $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$
 we focus on 2^k design
 $k = 0$
 fraction: $\{1, \dots, k-1\}$
 $2^{\text{resolution}}: \{\text{III}, \text{IV}, \text{V}, \dots\}$

Ex: reactor data

$I = ABCDE$ defining relation $\leftarrow$ we do row $2, 3, 5, \dots, 32$

$2^4 \rightarrow E = ABCD$ generator $\leftarrow$

Resolution: II

Alias:

$$\begin{aligned} A \cdot I &= BCDE \\ D \cdot I &= ACDE \end{aligned}$$

$\uparrow$
 Either think to start with full factorial in A, B, C, D and then add $E = ABCD$, or just do the runs with $ABCDE = I = I$.

$$AB \cdot I = AB \cdot ABCDE = CDE$$

$\vdots$

$\rightarrow$ see R-code L20.R

Interpretation of confounding

$$1) y = 70 + 4z_1 + 10z_2 + 1z_3 + 2z_{12} + 3z_{23} + 2z_{123}$$

$$2) C = AB, A = BC, B = AC, ABC = I$$

$$3) y = 72 + 7z_1 + 11z_2 + 3z_3$$

$$4) \begin{array}{l} \hat{A} = 2 \cdot \hat{\beta}_1 = 2 \cdot 7 \Rightarrow \hat{\beta}_1 = 7 \\ \hat{BC} = 14 \end{array} \Rightarrow \hat{\beta}_{23} = 7 \left. \vphantom{\begin{array}{l} \hat{A} = 2 \cdot \hat{\beta}_1 = 2 \cdot 7 \Rightarrow \hat{\beta}_1 = 7 \\ \hat{BC} = 14 \end{array}} \right\} \begin{array}{l} \text{but really} \\ \hat{A} = 8, \hat{BC} = 6 \end{array}$$

We think we estimate A , but really estimate

$$\hat{A} + \hat{BC} = 14.$$

Finally: bicycle example

Have 7 factors and perform 8 experiments $\Rightarrow$

2^{7-4} design. Think: full factorization A, B, C and

add generators $\begin{array}{l} D = \\ E = \\ F = \\ G = \end{array}$

see L20.2 for R-code.