$f(x)$

Let X_1 and X_2 be two (continuous) RVs, and $f(x_1, x_2)$ be the joint pdf and $f_1(x_1)$ and $f_2(x_2)$ be the marginal pdfs, and C is a copula. What is true?

A $f(x_1, x_2) = f_1(x_1) \cdot f_2(x_2)$

B $f(x_1, x_2)$ is found from $f_1(x_1)$ and $f_2(x_2)$ alone

C $f_1(x_1)$ is found from $f(x_1, x_2)$

D $f(x_1, x_2) = C(f_1(x_1), f_2(x_2))$

Mean of sum

$\boldsymbol{X}$ and $\boldsymbol{Y}$ are two bivariate random vectors with $\mathrm{E}(\boldsymbol{X}) = (1,2)^T$ and $\mathrm{E}(\boldsymbol{Y}) = (2,0)^T$. What is $\mathrm{E}(\boldsymbol{X}+\boldsymbol{Y})$?

A $(1.5, 1)^T$

B $(3, 2)^T$

C $(-1, 2)^T$

D $(1, -2)^T$

Mean of linear combination

$\boldsymbol{X}$ is a 2-dimensional random vector with $\mathrm{E}(\boldsymbol{X}) = (2, 5)^T$, and $\boldsymbol{b} = (0.5, 0.5)^T$ is a constant vector. What is $\mathrm{E}(\boldsymbol{b}^T\boldsymbol{X})$?

A 3.5

B 7

C 2

D 5

Covariance

$\boldsymbol{X}$ is a p-dimensional random vector with mean $\boldsymbol{\mu}$. Which of the following defines the covariance matrix?

A $E[(\boldsymbol{X}-\boldsymbol{\mu})^T(\boldsymbol{X}-\boldsymbol{\mu})]$

B $E[(\boldsymbol{X}-\boldsymbol{\mu})(\boldsymbol{X}-\boldsymbol{\mu})^T]$

C $E[(\boldsymbol{X}-\boldsymbol{\mu})(\boldsymbol{X}-\boldsymbol{\mu})]$

D $E[(\boldsymbol{X}-\boldsymbol{\mu})^T(\boldsymbol{X}-\boldsymbol{\mu})^T]$

Mean of linear combinations

$\boldsymbol{X}$ is a p-dimensional random vector with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. $\boldsymbol{C}$ is a constant matrix. What is then the mean of the k-dimensional random vector $\boldsymbol{Y} = \boldsymbol{C}\boldsymbol{X}$?

A $\boldsymbol{C}\boldsymbol{\mu}$

B $\boldsymbol{C}\boldsymbol{\Sigma}$

C $\boldsymbol{C}\boldsymbol{\mu}\boldsymbol{C}^T$

D $\boldsymbol{C}\boldsymbol{\Sigma}\boldsymbol{C}^T$

Covariance of linear combinations

$\boldsymbol{X}$ is a p-dimensional random vector with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. $\boldsymbol{C}$ is a constant matrix. What is then the covariance of the k-dimensional random vector $\boldsymbol{Y} = \boldsymbol{C}\boldsymbol{X}$?

A $\boldsymbol{C}\boldsymbol{\mu}$

B $\boldsymbol{C}\boldsymbol{\Sigma}$

C $\boldsymbol{C}\boldsymbol{\mu}\boldsymbol{C}^T$

D $\boldsymbol{C}\boldsymbol{\Sigma}\boldsymbol{C}^T$

Correlation

$\boldsymbol{X}$ is a 2-dimensional random vector with covariance matrix

$$\boldsymbol{\Sigma} = \begin{bmatrix} 4 & 0.8 \\ 0.8 & 1 \end{bmatrix}$$

Then the correlation between the two elements of $\boldsymbol{X}$ are:

A 0.10

B 0.25

C 0.40

D 0.80

Symmetric positive definite matrix

Which of the following is not correct for a symmetric positive definite matrix?

A The trace equals the rank of the matrix.

B The determinant is positive.

C The trace is the sum of the eigenvalues.

D All the eigenvalues are positive.

PCA interpretation

Data set: student's score on a Math test, a Physics test, a Reading comprehension test, and a Vocabulary test.

First PC represents overall academic ability,
second PC represents a contrast between quantitative ability and verbal ability.

What loadings would be consistent with that interpretation?

- A (0.5,0.5,0.5,0.5) and (0.71,0.71,0,0)
- B (0.5,0.5,0.5,0.5) and (0.5,0.5,-0.5,-0.5)
- C (0.71,0.71,0,0) and (0,0,0.71,-0.71)
- D (0.71,0,-0.71,0) and (0 ,0.71,0,-0.71)

Correct?

Are you sure you want to read the correct answers? Maybe try first? The answers are explained on the next two slides.

Answers

1. C: We go from joint to marginal distribution by integration. The product of marginals equal the joint only for independent variables. We need information on the dependency structure to construct a joint from marginals, and that is what is done with the copula - but the formula is based on the cumulative distribution functions.
2. B: Mean of sum $(1,2)^T + (2,0)^T = (3,2)^T$.
3. A: Mean of linear combination $(0.5, 0.5)^T(2,5) = 3.5$.
4. B: Covariance matrix defined as $E\{(\boldsymbol{X} - \boldsymbol{\mu})(\boldsymbol{X} - \boldsymbol{\mu})^T\}$. This was the only formula that gave a $p \times p$ matrix. A gave a scalar and C and D did not match in dimensions.

Answers

5. A: $\boldsymbol{C}\boldsymbol{\mu}$ is the mean of $\boldsymbol{Y} = \boldsymbol{C}\boldsymbol{X}$.
6. D: $\boldsymbol{C}\boldsymbol{\Sigma}\boldsymbol{C}^T$ is the covariance matrix of $\boldsymbol{Y} = \boldsymbol{C}\boldsymbol{X}$.
7. C: Correlation is 0.40 since covariance was 0.8 and variances 4 and 1.
8. A: NOT true for a symmetric positive definite matrix: the trace is in general not equal to the rank - but it is for idempotent symmetric matrices.
9. B: average means equal weight for all values, difference between quantitative and verbal means opposite signs for quantitative (maths and physics) and verbal (reading and vocabular).