Multivariate normal pdf

The probability density function is $\left(\frac{1}{2\pi}\right)^{\frac{p}{2}} \det(\boldsymbol{\Sigma})^{-\frac{1}{2}} \exp\{-\frac{1}{2}Q\}$ where Q is

A $(\boldsymbol{x} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\boldsymbol{x} - \boldsymbol{\mu})$

B $(\boldsymbol{x} - \boldsymbol{\mu}) \boldsymbol{\Sigma} (\boldsymbol{x} - \boldsymbol{\mu})^T$

C $\boldsymbol{\Sigma} - \boldsymbol{\mu}$

Trivariate normal pdf

What graphical form has the solution to $f(\boldsymbol{x}) = \text{constant}$?

A Circle

B Parabola

C Ellipsoid

D Bell shape

Multivariate normal distribution

$\boldsymbol{X}_p \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, and $\boldsymbol{C}$ is a $k \times p$ constant matrix.
$\boldsymbol{Y} = \boldsymbol{C}\boldsymbol{X}$ is

A Chi-squared with k degrees of freedom

B Multivariate normal with mean $k\boldsymbol{\mu}$

C Chi-squared with p degrees of freedom

D Multivariate normal with mean $\boldsymbol{C}\boldsymbol{\mu}$

Independence

Let $\boldsymbol{X} \sim N_3(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, with $\boldsymbol{\Sigma} = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 3 & 1 \\ 0 & 2 & 5 \end{bmatrix}$. Which two variables are independent?

A X_1 and X_2

B X_1 and X_3

C X_2 and X_3

D None – but two are uncorrelated.

Constructing independent variables?

Let $\boldsymbol{X} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. How can I construct a vector of independent standard normal variables from $\boldsymbol{X}$?

A $\boldsymbol{\Sigma}(\boldsymbol{X} - \boldsymbol{\mu})$

B $\boldsymbol{\Sigma}^{-1}(\boldsymbol{X} + \boldsymbol{\mu})$

C $\boldsymbol{\Sigma}^{-\frac{1}{2}}(\boldsymbol{X} - \boldsymbol{\mu})$

D $\boldsymbol{\Sigma}^{\frac{1}{2}}(\boldsymbol{X} + \boldsymbol{\mu})$

Conditional distribution: mean

$\boldsymbol{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$ is a bivariate normal random vector.
What is true for the conditional mean of
X_2 given $X_1 = x_1$?

A Not a function of x_1

B A linear function of x_1

C A quadratic function of x_1

Conditional distribution: variance

$\boldsymbol{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$ is a bivariate normal random vector.

What is true for the conditional variance of X_2 given $X_1 = x_1$?

A Not a function of x_1

B A linear function of x_1

C A quadratic function of x_1

Estimator for mean

$\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_n$ is a random sample from $N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. What is the distribution of the estimator $\bar{\boldsymbol{X}}$ for the mean?

A $N_n(\boldsymbol{\mu}, \boldsymbol{\Sigma})$

B $N_p(\boldsymbol{\mu}, \frac{1}{n}\boldsymbol{\Sigma})$

C χ^2_p

D χ^2_n

Unbiased estimators

$\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_n$ is a random sample of size n of a p-dimensional random vector. An unbiased estimator for the covariance matrix $\boldsymbol{\Sigma}$ is.

A $\frac{1}{n}\sum_{j=1}^{n}(\boldsymbol{X}_j - \bar{\boldsymbol{X}})(\boldsymbol{X}_j - \bar{\boldsymbol{X}})^T$

B $\frac{1}{n-1}\sum_{j=1}^{n}(\boldsymbol{X}_j - \bar{\boldsymbol{X}})(\boldsymbol{X}_j - \bar{\boldsymbol{X}})^T$

C $\frac{1}{n}\sum_{j=1}^{n}(\boldsymbol{X}_j - \bar{\boldsymbol{X}})^T(\boldsymbol{X}_j - \bar{\boldsymbol{X}})$

D $\frac{1}{n-1}\sum_{j=1}^{n}(\boldsymbol{X}_j - \bar{\boldsymbol{X}})^T(\boldsymbol{X}_j - \bar{\boldsymbol{X}})$

Distribution of quadratic form

$\boldsymbol{X} \sim N_p(\mathbf{0}, \boldsymbol{I})$, and $\boldsymbol{R}$ is a symmetric and idempotent matrix with rank r. What is the distribution of $\boldsymbol{X}^T\boldsymbol{R}\boldsymbol{X}$?

A $N_p(\boldsymbol{\mu}, r\boldsymbol{I})$

B $N_r(\mathbf{0}, \boldsymbol{I})$

C χ^2_r

D χ^2_p

Correct?

Are you sure you want to read the correct answers? Maybe try first? The answers are explained on the next two slides.

Answers

1. A: exponent quadratic form is $(\boldsymbol{x} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\boldsymbol{x} - \boldsymbol{\mu})$.
2. C: contours are ellipsoids in general. In two dimensions we have ellipses. For two dimensions and equal variance and correlation 0 we have circles.
3. D: linear combinations of mvN are also mvN.
4. B: $\mathrm{Cov}(X_1, X_3) = 0$ and X_1 and X_3 are thus independent.
5. C: The Mahlanobis transform is $\boldsymbol{\Sigma}^{-\frac{1}{2}} (\boldsymbol{X} - \boldsymbol{\mu})$.

Answers

6. B: Conditional mean is linear in x_1, which will be very useful when we start with multiple linear regression.
7. A: Conditional variance (covariance) is not a function of x_1.
8. B: The mean is also mvN with mean $\boldsymbol{\mu}$ and covariance $\frac{1}{n}\boldsymbol{\Sigma}$.
9. B: $\frac{1}{n-1}\sum_{j=1}^{n}(\boldsymbol{X}_j - \bar{\boldsymbol{X}})(\boldsymbol{X}_j - \bar{\boldsymbol{X}})^T$ is the unbiased estimator for $\boldsymbol{\Sigma}$. Observe the $(n-1)$ and that the dimension is $p \times p$ (to place the transpose). Not a quadratic form.
10. C: Quadratic form is related to χ^2.