Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4267 Linear statistical models 2020 spring
Compulsory exercises 4

Problem 1

a) Not every matrix is a covariance matrix of a random vector. Why? Is any matrix a covariance matrix of two random vectors? Justify your answer.

Problem 2

Let $X = (X_1, X_2, X_3)^T$ be a trivariate random vector. Suppose that all three components X_1, X_2, X_3 are normal.

a) Is X (always) a trivariate normal random vector?

b) Suppose that X_1, X_2, X_3 are independent? Does this imply that X is a trivariate normal random vector?

c) Suppose that X_1 and bivariate vector (X_2, X_3) are independent. Does this imply that X is a trivariate normal random vector?

Justify your answers.

Problem 3

Suppose that there are two random samples $X_1, ..., X_n$ and $Y_1, ..., Y_m$. All observations

$$X_1, ..., X_n, Y_1, ..., Y_m$$

are independent. X-s are drawn from a normal distribution with unknown expectation β_0 and variance 1. Y-s satisfy the linear regression model

$$Y_i = \beta_0 + \beta_1 y_i + \epsilon_i, \;\; i = 1, ..., m,$$

where β_0, β_1 are unknown parameters (β_0 is the same as for X-s), $y_1, ..., y_m$ are known numbers, independent errors $\epsilon_1, ..., \epsilon_m$ have the standard normal distribution.

a) Parameters β_0 and β_1 are estimated on the basis of the two samples, X-s and Y-s. Which estimator you could propose? Explain why you think your estimator is reasonable.

Problem 4

A multiple linear regression model is considered. It is assumed that

$$Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \varepsilon_i,$$

where ε-s are independent and have the same normal distribution with zero expectation and unknown variance σ^2. 30 measurements are made, i.e. $i = 1, 2, ..., 30$. The explanatory variables take the following values: $x_{i1} = 1$ for $11 \leq i \leq 20$ and 0 otherwise, $x_{i2} = 1$ for $21 \leq i \leq 30$ and 0 otherwise. Some responses Y-s and residuals $\hat{\varepsilon}$-s are given in the tables below. The coefficient of determination and the total sum of squares are equal to 0.4246 and 42.6397.

Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9	Y_{10}	Y_{11}	Y_{12}	Y_{13}	Y_{14}	Y_{15}
3.7	1.4			3.6		1.9		2.0			2.5			
Y_{16}	Y_{17}	Y_{18}	Y_{19}	Y_{20}	Y_{21}	Y_{22}	Y_{23}	Y_{24}	Y_{25}	Y_{26}	Y_{27}	Y_{28}	Y_{29}	Y_{30}
	3.8				3.9				5.7		4.2			3.7

$\hat{\varepsilon}_1$	$\hat{\varepsilon}_2$	$\hat{\varepsilon}_3$	$\hat{\varepsilon}_4$	$\hat{\varepsilon}_5$	$\hat{\varepsilon}_6$	$\hat{\varepsilon}_7$	$\hat{\varepsilon}_8$	$\hat{\varepsilon}_9$	$\hat{\varepsilon}_{10}$	$\hat{\varepsilon}_{11}$	$\hat{\varepsilon}_{12}$	$\hat{\varepsilon}_{13}$	$\hat{\varepsilon}_{14}$	$\hat{\varepsilon}_{15}$
	-1.1		0.1				1.3	-0.5			-0.8		-1.2	
$\hat{\varepsilon}_{16}$	$\hat{\varepsilon}_{17}$	$\hat{\varepsilon}_{18}$	$\hat{\varepsilon}_{19}$	$\hat{\varepsilon}_{20}$	$\hat{\varepsilon}_{21}$	$\hat{\varepsilon}_{22}$	$\hat{\varepsilon}_{23}$	$\hat{\varepsilon}_{24}$	$\hat{\varepsilon}_{25}$	$\hat{\varepsilon}_{26}$	$\hat{\varepsilon}_{27}$	$\hat{\varepsilon}_{28}$	$\hat{\varepsilon}_{29}$	$\hat{\varepsilon}_{30}$
	0.5	0.1				0.5		0.5	1.4	-0.5				

a) Perform a test for

$$H_0 : \beta_1 = \beta_2 + 1$$

vs.

$$H_1 : \beta_1 \neq \beta_2 + 1.$$

The significance level is 0.05. You should calculate a test statistic and use its distribution under the null hypothesis to arrive at your conclusion.