Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4267 Linear statistical models
Recommended exercises 6

Problem 1 Orthogonally projecting matrices

An $n \times p$ matrix R always maps onto its column space $\operatorname{Col} R$, in the sense that $R\boldsymbol{y} \in \operatorname{Col} R$ for all $\boldsymbol{y} \in \mathbb{R}^p$, and the map is onto, since any linear combination of columns of R can be written $R\boldsymbol{y}$.

If R is $n \times n$, it makes sense to ask when R projects orthogonally onto $\operatorname{Col} R$, that is, when is $R\boldsymbol{y}$ the orthogonal projection of $\boldsymbol{y}$ onto $\operatorname{Col} R$ for all $\boldsymbol{y} \in \mathbb{R}^n$? By the orthogonal decomposition theorem (the projection theorem), this is equivalent to $\boldsymbol{y} - R\boldsymbol{y} = (I - R)\boldsymbol{y}$ being in the orthogonal complement of $\operatorname{Col} R$ for all $\boldsymbol{y}$, that is, $0 = (R\boldsymbol{z})^{\mathrm{T}}(I - R)\boldsymbol{y} = \boldsymbol{z}^{\mathrm{T}} R^{\mathrm{T}}(I - R)\boldsymbol{y}$ for all $\boldsymbol{y}$, $\boldsymbol{z} \in \mathbb{R}^n$, which is the case if and only if $R^{\mathrm{T}}(I - R) = O$, an $n \times n$ zero matrix.

Show that $R^{\mathrm{T}}(I - R) = O$ if and only if R is symmetric and idempotent.

(We have also seen in the lectures that if X is $n \times p$ and $\operatorname{rank} X = p$, then the symmetric and idempotent matrix $H = X(X^{\mathrm{T}}X)^{-1}X^{\mathrm{T}}$ projects orthogonally onto $\operatorname{Col} X = \operatorname{Col} H$.)

Problem 2 Period of swing of pendulum

Exam TMA4267 2015 spring, Problem 1 – links in bokmål, nynorsk and English.

To get acces to data:

```
attach(read.table(
  "https://www.math.ntnu.no/emner/TMA4267/2018v/pendulum.txt"
))
```

Problem 3 Galápagos species

Exam TMA4267 2014 spring, Problem 2 – links in bokmål, nynorsk and English.