The Covariance Matrix $\boldsymbol{\Sigma}$

Definition: $Cov[\mathbf{X}] = E\left[\left(\boldsymbol{X} - \boldsymbol{\mu}\right)\right]\left(\boldsymbol{X} - \boldsymbol{\mu}\right)^T] = \boldsymbol{\Sigma}$

$$Cov[\boldsymbol{AX} + \boldsymbol{b}] = \boldsymbol{A\Sigma A}^T$$

$$Cov[\boldsymbol{AX}, \boldsymbol{BY}] = \boldsymbol{A\Sigma}_{xy}\boldsymbol{B}^T$$

$$Cov[\boldsymbol{X} + \boldsymbol{Y}, \boldsymbol{Z}] = Cov[\boldsymbol{X}, \boldsymbol{Z}] + Cov[\boldsymbol{Y}, \boldsymbol{Z}]$$

$$Cov[\boldsymbol{X} + \boldsymbol{Y}] = Cov[\boldsymbol{X}] + Cov[\boldsymbol{X}, \boldsymbol{Y}] + Cov[\boldsymbol{Y}, \boldsymbol{X}] + Cov[\boldsymbol{Y}]$$

$$Cov[\boldsymbol{a}^T\boldsymbol{X}] = \boldsymbol{a}^T\boldsymbol{\Sigma a}$$

$$\boldsymbol{\Sigma} = \sum_{i=1}^{p} \lambda_i \boldsymbol{e}_i \boldsymbol{e}_i^T = \boldsymbol{P\Lambda P}^T$$