

Mahalanobis transformation

$\mathbf{X}$ multivariate, $E[\mathbf{X}] = \boldsymbol{\mu}$, $\text{Cov}[\mathbf{X}] = \boldsymbol{\Sigma}$

$$\mathbf{Y} = \boldsymbol{\Sigma}^{-1/2} (\mathbf{X} - \boldsymbol{\mu}), E[\mathbf{Y}] = \mathbf{0}, \text{Cov}[\mathbf{Y}] = \mathbf{I}.$$

Principal components

$\boldsymbol{\Sigma}_X = \mathbf{P}\boldsymbol{\Lambda}\mathbf{P}^T$. Principal components $\mathbf{Y} = \mathbf{P}^T \mathbf{X}$, $\text{Cov}[\mathbf{Y}] = \boldsymbol{\Lambda}$,
 $\text{tr}[\boldsymbol{\Sigma}] = \text{tr}[\boldsymbol{\Lambda}]$.

$\mathbf{Z}$ Standardized $\mathbf{X} \Rightarrow \text{Cov}[\mathbf{Z}] = \text{Cor}[\mathbf{X}] = \boldsymbol{\rho}$

$$\rho_{Y_i, X_k} = \frac{\sqrt{\lambda_i} e_{ik}}{\sqrt{\sigma_{kk}}}, \text{eigenvector-eigenvalues pairs from } \boldsymbol{\Sigma}.$$

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Interpretation: correlations, eigenvectors, eigenvalues, screeplot, biplots.