Rank of products of matrices

$\boldsymbol{A}$ and $\boldsymbol{B}$ matrices, $\boldsymbol{AB}$ defined. Then $rank(\boldsymbol{AB}) \leq \min(rank(\boldsymbol{A}), Rank(\boldsymbol{B}))$.

Rank and eigenvalues

$\boldsymbol{A}_{n \times n}$ symmetric implies $rank(\boldsymbol{A})$ equals the number of nonzero eigenvalues.

$\boldsymbol{A}_{n \times n}$ symmetric and idempotent with rank r implies *r* eigenvalues are 1 and *n-r* are zero.

$\boldsymbol{A}_{n \times n}$ symmetric and idempotent implies $tr(\boldsymbol{A}) = rank(\boldsymbol{A})$.

Independence of quadratic forms

$\boldsymbol{A}$ and $\boldsymbol{B}$ symmetric and idempotent with $\boldsymbol{AB}=\boldsymbol{0}$ and $\boldsymbol{X} \sim N_n(\boldsymbol{\mu}, \sigma^2 \boldsymbol{I})$. Then $(\boldsymbol{X}-\boldsymbol{\mu})^T \boldsymbol{A}(\boldsymbol{X}-\boldsymbol{\mu})$ and $(\boldsymbol{X}-\boldsymbol{\mu})^T \boldsymbol{B}(\boldsymbol{X}-\boldsymbol{\mu})$ are independent.

Distribution of quadratic forms

$\boldsymbol{A}_{n \times n}$ symmetric and idempotent with rank r. $\boldsymbol{X} \sim N_n(\boldsymbol{\mu}, \sigma^2 \boldsymbol{I})$.

Then $\dfrac{(\boldsymbol{X}-\boldsymbol{\mu})^T \boldsymbol{A}(\boldsymbol{X}-\boldsymbol{\mu})}{\sigma^2} \sim \chi^2(r)$.

Multiple linear regression model.

$\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon}$, where $E[\boldsymbol{\varepsilon}] = \boldsymbol{0}$, and $Cov(\boldsymbol{\varepsilon}) = \sigma^2 \boldsymbol{I}$

Least squares estimators

Coefficients: $\hat{\boldsymbol{\beta}} = \left(\boldsymbol{X}^T \boldsymbol{X} \right)^{-1} \boldsymbol{X}^T \boldsymbol{Y}$

Model: $\hat{\boldsymbol{Y}} = \boldsymbol{X} \hat{\boldsymbol{\beta}}$

Residuals: $\boldsymbol{Y} - \hat{\boldsymbol{Y}}$

Properties

$E\left[\hat{\boldsymbol{\beta}} \right] = \boldsymbol{\beta}$, $Cov\left[\hat{\boldsymbol{\beta}} \right] = \sigma^2 \left(\boldsymbol{X}^T \boldsymbol{X} \right)^{-1}$