Recall example 1 2^3 experiment (2 runs, $n=16$)

A : temperature

B : Concentration

C : pH

Now: Can only do 4 exp. (half fraction)

$ABC = 1$

Note $A = BC$ ← column for A = column for BC

$B = AC$ "aliasing"

$C = AB$

Example 2^{4-1} design

4 factors, $2^4 = 16$ factor combinations

$2^{4-1} \rightarrow 2^3 = 8$ experiments

Effects: A B C D

AB AC AD BC BD CD

ABC ABD BCD

ABCD

only do experiments with $ABCD = 1$

Which columns will be equal?

$ABCD = 1$

$\underbrace{A \cdot A}_{1}BCD = A$

$BCD = A$

$\underbrace{B \cdot B}_{=1}CD = B \cdot A$

$CD = BA$

And $\quad B \cdot ABCD = B \qquad ACD \cdot D = BD$

$\qquad ACD = B \qquad\qquad AC = BD \qquad$ etc.

here main effects aliased with 3-factor interactions

2-factor int. aliased with 2-factor int.

Consequence?

$\hat{A}^*$: estimate of main effect in fraction $ABCD = 1$

$$\hat{A}^* = \frac{1}{8} \sum_{\substack{A=1 \\ ABCD=1}} y \;-\; \frac{1}{8} \sum_{\substack{A=-1 \\ ABCD=1}} y$$

$\hat{A}$ and $\hat{BCD}$: estimates in full factorial

$$\hat{A} + \hat{BCD} = \left(\frac{1}{16} \sum_{A=1} y - \frac{1}{16} \sum_{A=-1} y \right) + \left(\frac{1}{16} \sum_{BCD=1} y - \frac{1}{16} \sum_{BCD=-1} y \right)$$

$$= \left(\frac{1}{16} \left[\sum_{\substack{A=1 \\ BCD=1}} y + \underbrace{\cancel{\sum_{\substack{A=1 \\ BCD=-1}} y}}_{ABCD=-1} \right] - \frac{1}{16} \left[\underbrace{\cancel{\sum_{\substack{A=-1 \\ BCD=1}} y}}_{ABCD=-1} + \sum_{\substack{A=-1 \\ BCD=-1}} y \right] \right)$$

$$+ \left(\frac{1}{16} \left[\sum_{\substack{BCD=1 \\ A=1}} y + \underbrace{\cancel{\sum_{\substack{BCD=1 \\ A=-1}} y}}_{ABCD=-1} \right] - \frac{1}{16} \left[\underbrace{\cancel{\sum_{\substack{BCD=-1 \\ A=1}} y}}_{ABCD=-1} + \sum_{\substack{BCD=-1 \\ A=-1}} y \right] \right)$$

$$= \frac{2}{16} \sum_{\substack{A=1 \\ BCD=1}} y \;-\; \frac{2}{16} \sum_{\substack{A=-1 \\ BCD=-1}} y \;=\; \hat{A}^*$$

So $\hat{A}^* = \hat{A} + \hat{BCD}$

if $\hat{BCD} \approx 0$ then $\hat{A}^* = \hat{A}$

In the 2^{4-1} experiment A is aliased with BCD (etc),
but ok if $BCD \approx 0$.

Example 3: Drug combinations

2^{6-1} fractional factorial

Main effects A B C D E F

Defining relation $ABCDE = F$ ← main effect aliased with 5-factor int.

Which effects are aliased?

$A \circ ABCDE = A \circ F$

$BCDE = AF$ ← 2-factor int. aliased with 4-factor int.

$B \cdot BCDE = B \cdot AF$

$CDE = BAF$ ← 3-factor int. aliased with 3-factor int.

(etc.)

Assumption 4-factor, 5-factor and 6-factor effects ≈ 0

↳ can use the 2^{6-1} experiment with defining relation $ABCDE = F$ to estimate main, 2-factor int., and pairwise sums of 3-factor int.

Resolution (see def. on slide)

1-factor aliased with 5-factor

$p = 1$ $\quad R - p = 5 \Rightarrow R = 6$

2-factor aliased with 4-factor

$p = 2$ $\quad R - p = 4 \Rightarrow R = 6$

3-factor aliased with 3-factor

$p = 3$ $\quad R - p = 3 \Rightarrow R = 6$

What if defining relation was $ABC = 1$

$$A \cdot ABC = A$$

$$BC = A$$

1-factor aliased with 2 factor

$p = 1$ $\quad R - p = 2 \Rightarrow R = 3.$

[Results, note center runs]

Sums of Squares

Recall $\quad SS_T = SS_R + SS_E$

$SS_T = \Sigma (y - \bar{y})^2 \qquad SS_E = \Sigma (y - \hat{y})^2$ residuals

$$SSR = n\hat{\beta}_1^2 + \underbrace{n\hat{\beta}_2^2}_{} + \ldots$$

2-level factorial

Variability explained by factor B in the model.

[note ABC I DEF]

Factor D: $n\hat{\beta}_2^2 = 32 \cdot (-0.141)^2 = 0.636..$

$$\frac{n\hat{\beta}_2^2}{SST} = 0.68$$ → 68% variability explained by factor D.

$$\frac{n \sum_{\text{main effects}} \hat{\beta}_j^2}{SS_T} = 0.815$$

$$\frac{n \sum_{\text{2-factor int}} \hat{\beta}_j^2}{SS_T} = 0.068$$

$$\frac{n \sum_{\text{3-factor}} \hat{\beta}_j}{SS_T} = 0.032$$

$$\frac{SSE}{SST} = 0.083$$

Plots:

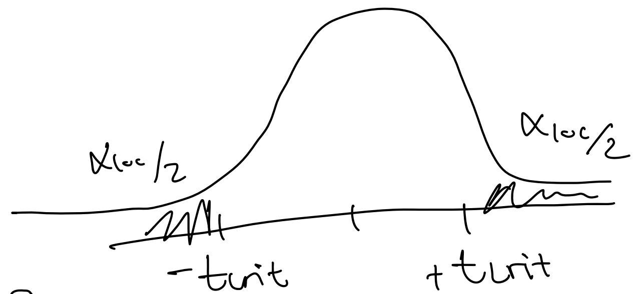
main effects: PH greatest effect, then conc., then temp.

Int. effects:

Pareto:

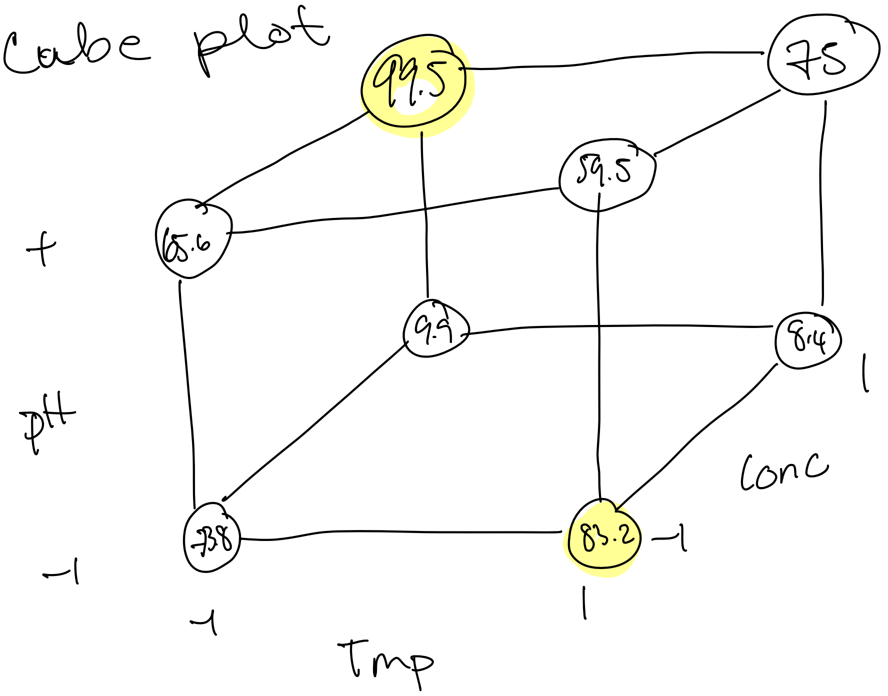
add critical value

T-test $\dfrac{\hat{\beta}_j}{\hat{se}(\hat{\beta}_j)} \overset{H_0}{\sim} \underbrace{t_{n-p}}_{16-8}$



critical $|2\hat{\beta}_j| \rightarrow t_{crit} \cdot \hat{se}(\beta_j) \cdot 2$

Cube plot



means for each combination

shows which factor comb. are favorable.

tmp -1, ph +1, conc +1

same sign.

we see interaction ph·conc here

tmp -1, ph -1, con -1