

TMA4267: Assignment 2

Chapter 4.4 Properties of the multivariate normal distribution

Exercises from the book

Practice working with properties of the multivariate normal distribution: 4.16 a, b, c, d, e

Theorem 4.4c.

- (a) Let $\mathbf{Y} = (Y_1, Y_2)^T$ be bivariate normal with mean $\boldsymbol{\mu} = (\mu_1, \mu_2)^T$ and covariance matrix $\Sigma = \begin{bmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{22} \end{bmatrix}$. Show that Y_1 and Y_2 are independent by showing that the joint moment generating function (mgf) of $\mathbf{Y}$ factors into the product of the mgfs of (univariate) Y_1 and Y_2 .
- (b) Explain how the result in corollary 2 follows from theorem 4.4c.

Theorem 4.4d.

- (a) In simple linear regression we assume that $E(Y|x) = \beta_0 + \beta_1 x$ and that $\text{Var}(Y|x) = \sigma^2$. Consider now the bivariate normal random vector $(X, Y)^T$ with mean $\boldsymbol{\mu} = (\mu_X, \mu_Y)^T$ and covariance matrix $\Sigma = \begin{bmatrix} \sigma_{XX} & \sigma_{XY} \\ \sigma_{YX} & \sigma_{YY} \end{bmatrix}$. Write down $E(Y|x)$ and $\text{Var}(Y|x)$ in terms of elements of $\boldsymbol{\mu}$ and Σ . Find the relationship between these expressions and the parameters β_0 , β_1 and σ^2 in a simple linear regression model.
- (b) Consider the example from Tuesday regarding the (empirical) joint and conditional distribution of summer temperatures in Oslo and Trondheim. Use the figure to discuss if these conditional densities agree with the assumption of a bivariate normal distribution.

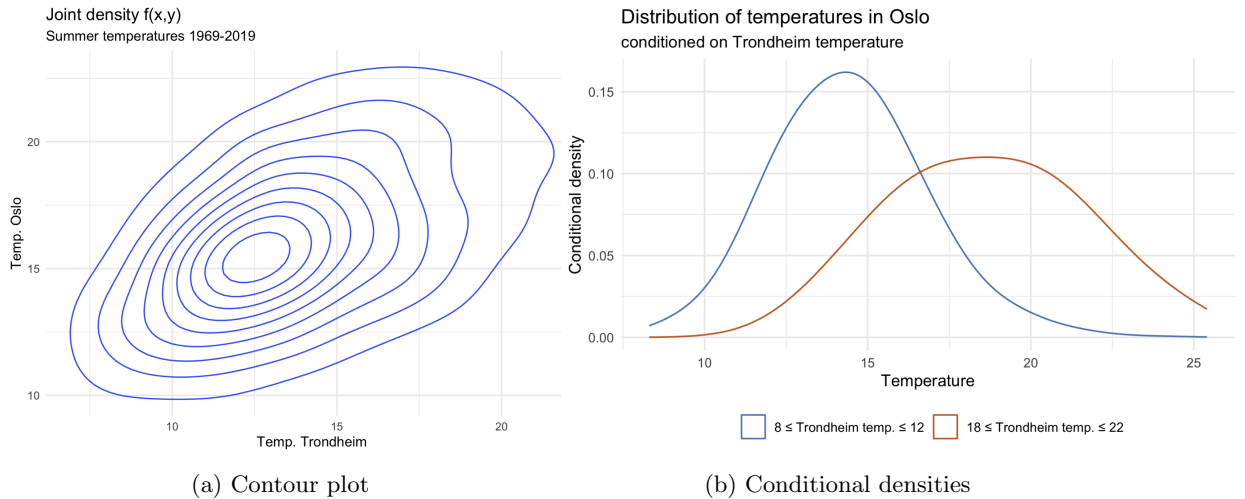


Figure 1: Summer temperatures in Trondheim and Oslo

Exam questions

Summer 2014

Let $\mathbf{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$ be a bivariate normal random vector with mean $\boldsymbol{\mu} = E(\mathbf{X}) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and covariance matrix

$$\Sigma = \text{Cov}(\mathbf{X}) = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 2 \end{pmatrix}.$$

Let

$$\mathbf{Y} = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}, \quad Y_1 = 3X_1 - 2X_2, \quad Y_2 = X_1 + X_2.$$

What is the distribution of $\mathbf{Y}$?

Let $Z = X_1 + aX_2$. How can you choose a so that Z and Y_2 are independent?

Justify your answer.

If you have time for more: Spring 2011, problem 1

An exam consists of two tasks. For each task a score is given, and the final mark is decided from the sum of the scores for each task. Let X_1 be the score on task 1 and let X_2 be the score on task 2. For each student we assume that

$$X_1 \sim N(\mu, \sigma^2), \quad X_2 \sim N(k\mu, k^2\sigma^2),$$

and that

$$\mathbf{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$$

is bivariate normal. Let the correlation between X_1 and X_2 be ρ .

a) Find the distribution of the random vector

$$\mathbf{Y} = \begin{pmatrix} X_1 + X_2 \\ X_1 \end{pmatrix}.$$

Explain your answer. Find the expectation of $\mathbf{Y}$, $\boldsymbol{\mu}_Y$, and the covariance matrix of $\mathbf{Y}$, Σ_Y .

b) Find

$$E(X_1 + X_2 \mid X_1 = x_1).$$

What is the expected final score for a student that has a score of one point more than expected on the first task when $k = 4$ and $\rho = \frac{1}{2}$?