

TMA4267 Linear statistical models

Spring 2026

Thea Bjørnland

We continue Part 3

Tentative plan

Week	Tuesday	Thursday
15		Repetition + ANOVA
16	ANOVA / DOE	ANOVA / DOE
17	"summary of everything"	Exam prep
18	Exam prep	
19	06.05.26 EXAM 15:00 - 19:00	

NB: With the reference group we decided to skip mandatory assignments for Part 3

Other messages

Exam May 6th (evening)

Bring blue booklet and simple calculator

No handwritten notes



A formula sheet will be available in Inspira.
The first draft available on our wikipages

Please note: The course content has been changed as of this semester, with a new book and new exercises. We therefore primarily recommend to spend time on this years material, and not only on previous exams.

Overall F test

Snapshot from formula sheet

$$F = \frac{\text{SSR}/k}{\text{SSE}/(n - k - 1)}.$$

Sums of squares: $\text{SST} = \text{SSR} + \text{SSE}$, where

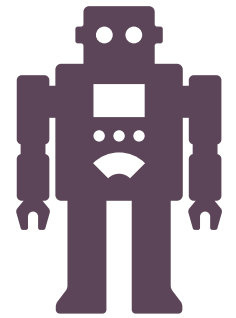
$$\text{SSE} = \mathbf{Y}^\top (\mathbf{I} - \mathbf{X}(\mathbf{X}^\top \mathbf{X})^{-1} \mathbf{X}^\top) \mathbf{Y} = \mathbf{Y}^\top (\mathbf{I} - \mathbf{H}) \mathbf{Y},$$

$$\text{SST} = \mathbf{Y}^\top \left(\mathbf{I} - \frac{1}{n} \mathbf{1} \mathbf{1}^\top \right) \mathbf{Y} = \mathbf{Y}^\top \left(\mathbf{I} - \frac{1}{n} \mathbf{J} \right) \mathbf{Y},$$

and

$$\text{SSR} = \mathbf{Y}^\top \left(\mathbf{H} - \frac{1}{n} \mathbf{J} \right) \mathbf{Y}.$$

Two-way (two- factor) anova

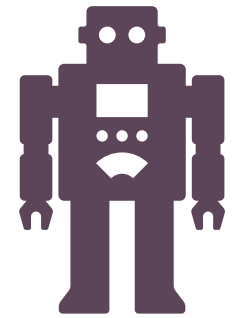


Machine operator example

```
> head(ds,8)
```

	time	operator	machine
[1,]	42.5	1	1
[2,]	39.8	1	2
[3,]	40.2	1	3
[4,]	41.3	1	4
[5,]	39.3	2	1
[6,]	40.1	2	2
[7,]	40.5	2	3
[8,]	42.2	2	4

Two-factor anova



Machine operator example

	(Intercept)	machine1	machine2	machine3	operator1	operator2	operator3	operator4	operator5
1	1	1	0	0	1	0	0	0	0
2	1	0	1	0	1	0	0	0	0
3	1	0	0	1	1	0	0	0	0
4	1	-1	-1	-1	1	0	0	0	0
5	1	1	0	0	0	1	0	0	0
6	1	0	1	0	0	1	0	0	0
7	1	0	0	1	0	1	0	0	0
8	1	-1	-1	-1	0	1	0	0	0
9	1	1	0	0	0	0	1	0	0
10	1	0	1	0	0	0	1	0	0
11	1	0	0	1	0	0	1	0	0
12	1	-1	-1	-1	0	0	1	0	0
13	1	1	0	0	0	0	0	1	0
14	1	0	1	0	0	0	0	1	0
15	1	0	0	1	0	0	0	1	0
16	1	-1	-1	-1	0	0	0	1	0
17	1	1	0	0	0	0	0	0	1
18	1	0	1	0	0	0	0	0	1
19	1	0	0	1	0	0	0	0	1
20	1	-1	-1	-1	0	0	0	0	1
21	1	1	0	0	-1	-1	-1	-1	-1
22	1	0	1	0	-1	-1	-1	-1	-1
23	1	0	0	1	-1	-1	-1	-1	-1
24	1	-1	-1	-1	-1	-1	-1	-1	-1

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	42.1208	0.2574	163.653	< 2e-16	***
machine1	-0.8208	0.4458	-1.841	0.08544	.
machine2	-0.7375	0.4458	-1.654	0.11882	
machine3	0.4458	0.4458	1.000	0.33313	
operator1	-1.1708	0.5755	-2.034	0.05999	.
operator2	-1.5958	0.5755	-2.773	0.01422	*
operator3	-0.8958	0.5755	-1.557	0.14042	
operator4	0.3292	0.5755	0.572	0.57583	
operator5	1.9292	0.5755	3.352	0.00437	**

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.261 on 15 degrees of freedom

Multiple R-squared: 0.7087, Adjusted R-squared: 0.5533

F-statistic: 4.561 on 8 and 15 DF, p-value: 0.005599

Partial F test

Snapshot from formula sheet

Partial F test: (* denotes reduced model)

$$F = \frac{(\text{SSR} - \text{SSR}^*)/q}{\text{SSE}/(n - k - 1)}.$$

Complete the ANOVA table:

```
> anova(fit)
```

Analysis of Variance Table

Response: time

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
machine	3	15.925	(2)	3.3388	0.047904	*
operator	(1)	42.087	8.4174	(3)	0.005328	**
Residuals	15	23.848	1.5899			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
> anova(fit)
```

```
Analysis of Variance Table
```

```
Response: time
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
machine	3	15.925	5.3082	3.3388	0.047904	*
operator	5	42.087	8.4174	5.2944	0.005328	**
Residuals	15	23.848	1.5899			

```
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```