

$X_{p \times 1}$ has mean μ_x and covariance matrix $\Sigma_x = \text{Cov}(X)$
 $= E[(X - \mu_x)(X - \mu_x)^T]$. Consider $Z = CX$.

Then $E(Z) = C\mu_x$ and $\text{Cov}(Z) = C\Sigma_x C^T$

PROOF:

$$\mu_z = E(Z) = E(CX) = E(CXI)$$

$$E(AXB)$$

where $B = I$

$$= C E(X) I = C E(X) = C\mu_x$$

$$\text{Cov}(Z) = E[(Z - \mu_z)(Z - \mu_z)^T]$$

$$= E[(CX - C\mu_x)(CX - C\mu_x)^T]$$

$$= E\left[C \underbrace{(X - \mu_x)(X - \mu_x)^T}_{\substack{\downarrow \\ \text{prop}}} C^T \right]$$

prop

$$\text{use } E(AXB) = A E(X) B$$

$$\begin{array}{c} \uparrow \quad \uparrow \quad \uparrow \\ C \quad X \quad C^T \end{array}$$

$$= C \underbrace{E[(X - \mu_x)(X - \mu_x)^T]}_{\text{def. of Cov}(X)} C^T = \underline{\underline{C \text{Cov}(X) C^T}}$$

def. of $\text{Cov}(X)$