

FORELESNING 10

Våren 2004

1. april

TMA4275 LEVETIDSANALYSE

Bo Lindqvist

Institutt for matematiske fag

NTNU

bo@math.ntnu.no

<http://www.math.ntnu.no/~bo/>

1

Insulate.MTW ***											
↓	C1	C2	C3	C4	C5-T	C6	C7	C8	C9	C10	C11
	Temp	ArrTemp	Plant	FailureT	Censor	Design	NewTemp	ArrNewT	NewPlant		
1	170	26,1865	1	343	F	80	80	32,8600	1		
2	170	26,1865	1	869	F	100	80	32,8600	2		
3	170	26,1865	1	244	C		100	31,0988	1		
4	170	26,1865	1	716	F		100	31,0988	2		
5	170	26,1865	1	531	F						
6	170	26,1865	1	738	F						
7	170	26,1865	1	461	F						
8	170	26,1865	1	221	F						
9	170	26,1865	1	665	F						
10	170	26,1865	1	384	C						
11	170	26,1865	2	394	C						
12	170	26,1865	2	369	F						
13	170	26,1865	2	366	F						
14	170	26,1865	2	507	F						
15	170	26,1865	2	461	F						
16	170	26,1865	2	431	F						
17	170	26,1865	2	479	F						
18	170	26,1865	2	106	F						
19	170	26,1865	2	545	F						
20	170	26,1865	2	536	F						
21	150	27,4242	1	2134	C						
22	150	27,4242	1	2746	F						
23	150	27,4242	1	2859	F						
24	150	27,4242	1	1826	C						

2

MINITAB Help

File Edit Bookmark Options Help

Help Topics Back Print << >> Glossary Exit

Example of Accelerated Life Testing

[main topic](#) [interpreting results](#) [session command](#) [see also](#)

Suppose you want to investigate the deterioration of an insulation used for electric motors. The motors normally run between 80 and 100° C. To save time and money, you decide to use accelerated life testing.

First you gather failure times for the insulation at abnormally high temperatures – 110, 130, 150, and 170° C – to speed up the deterioration. With failure time information at these temperatures, you can then extrapolate to 80 and 100° C. It is known that an Arrhenius relationship exists between temperature and failure time. To see how well the model fits, you will draw a probability plot based on the standardized residuals.

- 1 Open the worksheet INSULATE.MTW.
- 2 Choose **Stat > Reliability/Survival > Accelerated Life Testing**.
- 3 In **Variables/Start variables**, enter **FailureT**. In **Accelerating variable**, enter **Temp**.
- 4 From **Relationship**, choose **Arrhenius**.
- 5 Click **Censor**. In **Use censoring columns**, enter **Censor**, then click **OK**.
- 6 Click **Graphs**. In **Enter design value to include on plot**, enter **80**. Click **OK**.
- 7 Click **Estimate**. In **Enter new predictor values**, enter **Design**, then click **OK** in each dialog box.

Session window output

Regression with Life Data: FailureT versus Temp

Response Variable: FailureT

Censoring Information	Count
Uncensored value	66
Right censored value	14

Censoring value: Censor = C

Estimation Method: Maximum Likelihood
 Distribution: Weibull
 Transformation on accelerating variable: Arrhenius

3

Regression Table

Predictor	Coef	Standard Error	Z	P	95.0% Normal CI	
					Lower	Upper
Intercept	-15.1874	0.9862	-15.40	0.000	-17.1203	-13.2546
Temp	0.83072	0.03504	23.71	0.000	0.76204	0.89940
Shape	2.8246	0.2570			2.3633	3.3760

Log-Likelihood = -564.693

Anderson-Darling (adjusted) Goodness-of-Fit

At each accelerating level

Level	Fitted Model
110	*
130	*
150	*
170	*

Table of Percentiles

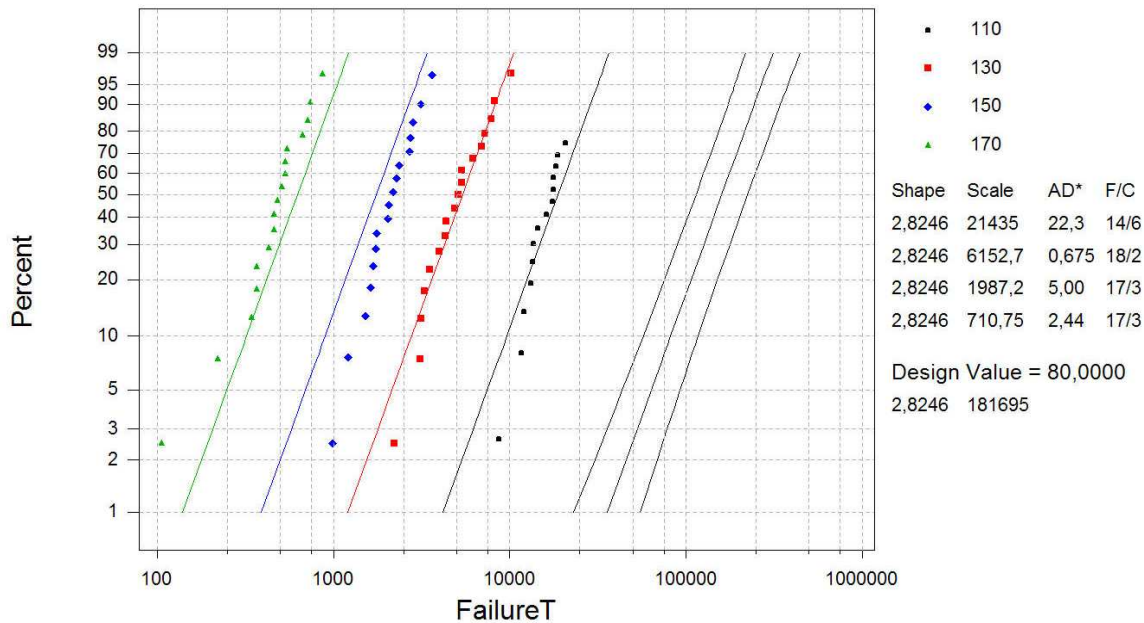
Percent	Temp	Percentile	Standard Error	95.0% Normal CI	
				Lower	Upper
50	80.0000	159584.5	27446.85	113918.2	223557.0
50	100.0000	36948.57	4216.511	29543.36	46209.94

4

Probability Plot (Fitted Arrhenius) for FailureT

Weibull Distribution - ML Estimates - 95,0% CI

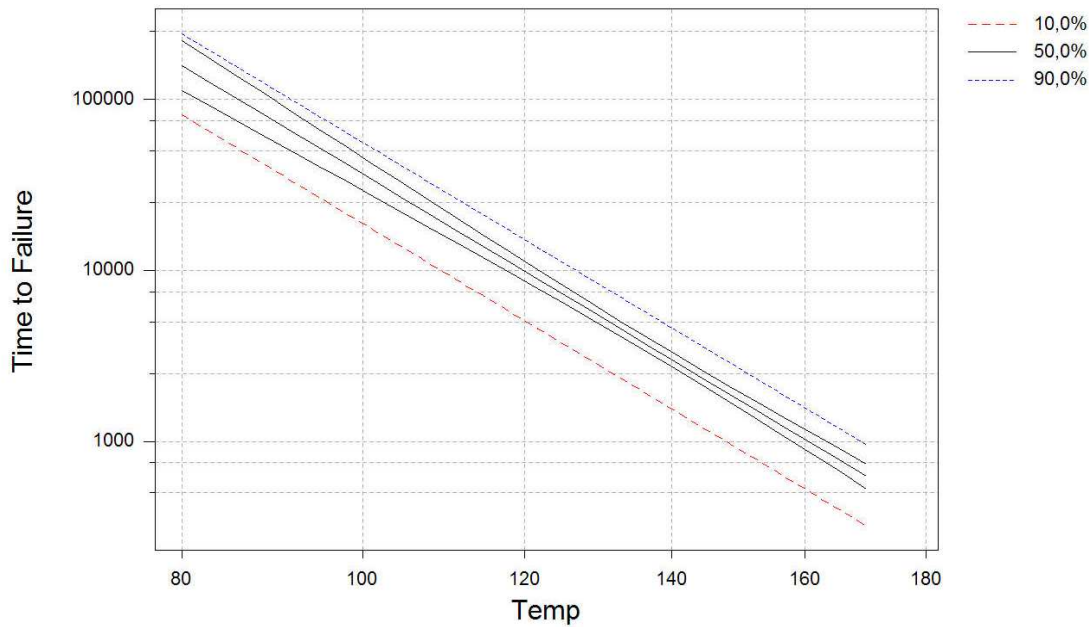
Censoring Column in Censor



Relation Plot (Fitted Arrhenius) for FailureT

Weibull Distribution - ML Estimates - 95,0% CI

Censoring Column in Censor



MED FAKTOR "PLANT" I TILLEGG:

Accelerated Life Testing

Responses are uncens/right censored data
 Responses are uncens/arbitrarily censored data

Variables/ Start variables: FailureT
End variables:
Freq. columns: (optional)

Accelerating var: Temp Relationship: Arrhenius

Second Variable
 Accelerating: Relationship: Linear
 Factor: Plant
 Include interaction term between variables

Assumed distribution: Weibull

Select Help Censor... Estimate... Graphs... Results... Options... Storage... OK Cancel

7

EKSAMEN MAI 2003, OPPGAVE 3:

En komponent antas under normalstress å ha overlevelsesfunksjon (reliability function) $R_0(t)$ og hasardfunksjon (sviktintensitet) $z_0(t)$.

Man ønsker å estimere påliteligheten av denne komponenttypen ved hjelp av akselerert levetidstesting. Dette gjøres ved å utsette komponenter for stress s , $0 \leq s < \infty$, og måle levetiden (eventuelt sensurerte levetider). Normalstress svarer til $s = 0$.

To modeller betraktes:

Modell 1: Proporsjonal hasardmodell. Under stress s har komponenten hasardfunksjon

$$z_s^{PH}(t) = z_0(t)g(s)$$

for en funksjon $g(s)$ med $g(0)=1$.

Modell 2: Akselerert levetidsmodell. Under stress s har komponenten overlevelsesfunksjon

$$R_s^{AL}(t) = R_0(\phi(s)t)$$

for en funksjon $\phi(s)$ med $\phi(0) = 1$.

8

(a) Forklar kort hva som er hensikten med akselerert levetidstesting. Hva er ideen bak de to modellene? Hva uttrykker de to funksjonene $g(s)$ og $\phi(s)$?

(b) La $R_s^{PH}(\cdot)$ være overlevelsesfunksjonen for en komponent under stress s i Modell 1. Vis at

$$R_s^{PH}(t) = R_0(t)^{g(s)}$$

La videre $z_s^{AL}(\cdot)$ være hasardfunksjonen for en komponent under stress s i Modell 2. Uttrykk $z_s^{AL}(\cdot)$ ved funksjonene $z_0(\cdot)$ og $\phi(\cdot)$.

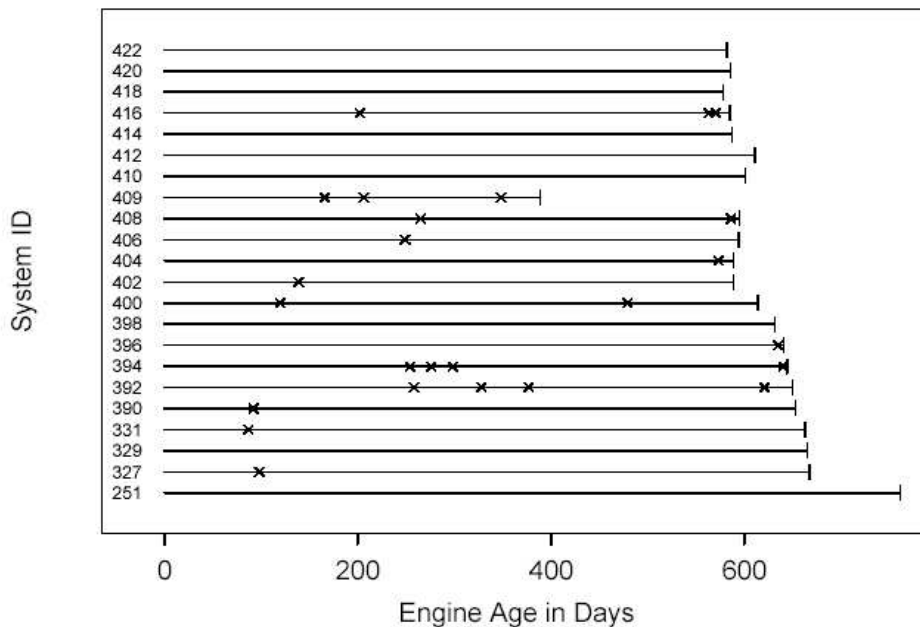
(c) Anta at komponentens levetid under normalstress er Weibull(α, θ), definert ved

$$R_0(t) = e^{-(t/\theta)^\alpha}$$

Vis at levetiden under stress $s > 0$ også er Weibull-fordelt under begge modellene. Hva blir parametrene i de tilsvarende Weibull-fordelingene?

I hvilken forstand kan vi si at Modell 1 og Modell 2 er ekvivalente ved Weibull-fordelte levetider?

Valve Seat Replacement Times Event Plot
(Nelson and Doganaksoy 1989)



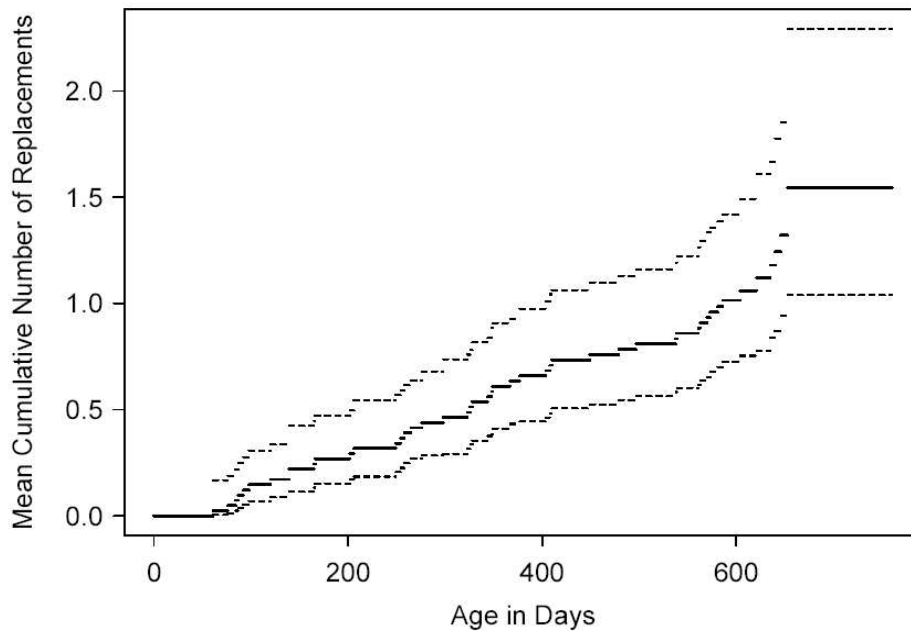
Valve Seat Replacement Times (Nelson and Doganaksoy 1989)

Data collected from valve seats from a fleet of 41 diesel engines (days of operation)

- Each engine has 16 valves
- Does the replacement rate increase with age?
- How many replacement valves will be needed in the future?
- Can valve life in these systems be modeled as a renewal process?

11

Estimate of Number of Valve Seat $\mu(t)$



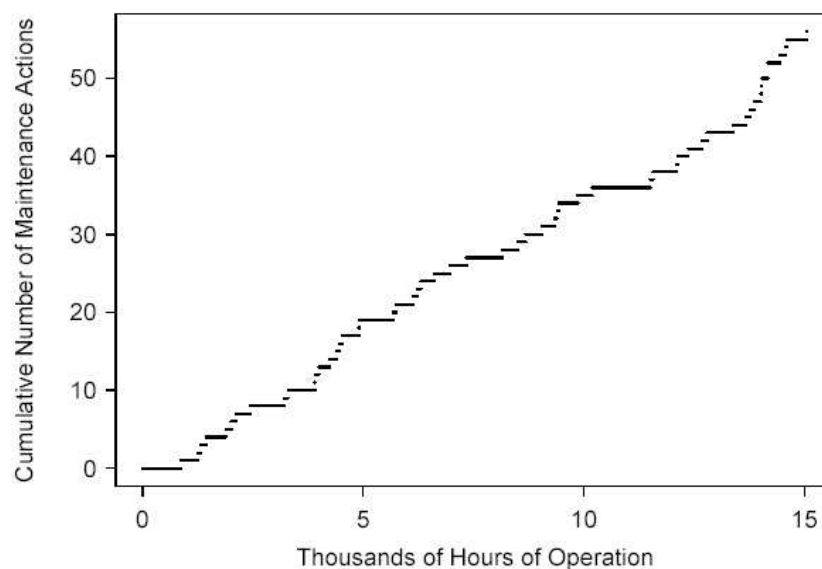
12

Times of Unscheduled Maintenance Actions for a USS Grampus Diesel Engine

- Unscheduled maintenance actions caused by failure of imminent failure.
- Unscheduled maintenance actions are inconvenient and expensive.
- Data available for 16,000 operating hours.
- Data from Lee (1980).
- Is the system deteriorating (i.e., are failures occurring more rapidly as the system ages)?
- Can the occurrence of unscheduled maintenance actions be modeled by an HPP?

13

Cumulative Number of Unscheduled Maintenance Actions Versus Operating Hours for a USS Grampus Diesel Engine Lee (1980)



14

The Likelihood for the NHPP - Single Unit

- With **interval** recurrence data.

Suppose that the unit has been observed for a period $(0, t_a]$ and the data are the number of recurrences d_1, \dots, d_m in the nonoverlapping intervals $(t_0, t_1], (t_1, t_2], \dots, (t_{m-1}, t_m]$ (with $t_0 = 0, t_m = t_a$).

$$\begin{aligned}
 L(\boldsymbol{\theta}) &= \Pr [N(t_0, t_1) = d_1, \dots, N(t_{m-1}, t_m) = d_m] \\
 &= \prod_{j=1}^m \Pr [N(t_{j-1}, t_j) = d_j] \\
 &= \prod_{j=1}^m \frac{[\mu(t_{j-1}, t_j; \boldsymbol{\theta})]^{d_j}}{d_j!} \exp [-\mu(t_{j-1}, t_j; \boldsymbol{\theta})] \\
 &= \prod_{j=1}^m \frac{[\mu(t_{j-1}, t_j; \boldsymbol{\theta})]^{d_j}}{d_j!} \times \exp [-\mu(t_0, t_a; \boldsymbol{\theta})]
 \end{aligned}$$

15

The Likelihood for the NHPP (Continued)

- If the number of intervals m increases and there are **exact** recurrences at $t_1 \leq \dots \leq t_r$ (here $r = \sum_{j=1}^m d_j$, $t_0 \leq t_1$, $t_r \leq t_a$), then using a limiting argument it follows that the likelihood in terms of the density approximation is

$$L(\boldsymbol{\theta}) = \prod_{j=1}^r \nu(t_j; \boldsymbol{\theta}) \times \exp [-\mu(0, t_a; \boldsymbol{\theta})]$$

- For simplicity, above we assumed that the intervals are contiguous. Obvious changes to the formula above give the likelihood when there are gaps among the intervals.
- In both cases (the interval data or exact recurrences data) the same methods used in Chapters 7, 8 can be used to obtain the ML estimate $\hat{\boldsymbol{\theta}}$ and confidence regions for $\boldsymbol{\theta}$ or functions of $\boldsymbol{\theta}$.

16

CONDITIONAL ROCOF BY MINIMAL REPAIR (NHPP) AND PERFECT REPAIR (RENEWAL PROCESS)

