FORELESNING 6
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TMA4275 LEVETIDSANALYSE

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RIGHT CENSORED DATA FOR "FORELESNING 6"
EXPONENTIAL MODEL

Row	C1	C2
1	0,35	1
2	0,50	0
3	0,75	0
4	1,00	1
5	1,30	1
6	1,80	1
7	3,00	0
8	3,15	0
9	4,85	0
10	5,50	1
11	5,50	0
12	6,25	0

Variable: C1 Censoring Information
Count Uncensored value 5 Right censored value 7 Censoring value: C2 = 0

Estimation Method: Maximum Likelihood
Distribution: Exponential

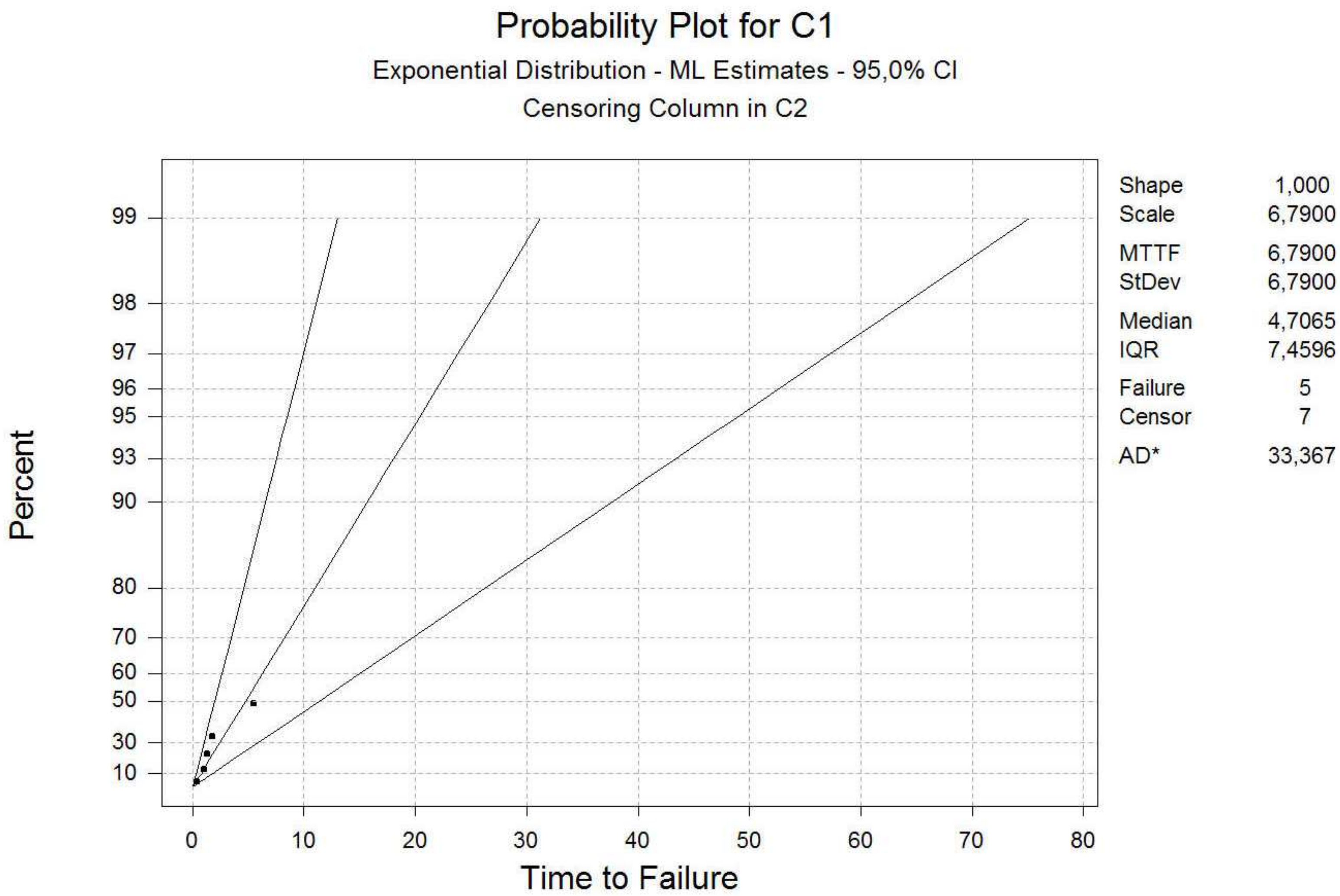
Parameter Estimates

		Standard	95,0% Normal CI	
Parameter	Estimate	Error	Lower	Upper
Shape	1,00000			
Scale	6,790	3,037	2,826	16,313

Log-Likelihood = -14,577

PLOT USING MINITAB 13
Plots the points

$$(t_i, -\ln(1 - F(t_i;\theta))$$



WEIBULL MODEL

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Variable:  C1

Censoring Information                 Count Uncensored value
5 Right censored value 7 Censoring value:  C2 = 0

Estimation Method:  Maximum Likelihood
Distribution:  Weibull

Parameter Estimates
                        Standard        95,0% Normal CI
Parameter   Estimate       Error        Lower        Upper
Shape         0,9780      0,3694       0,4665       2,0504
Scale         6,880       3,517        2,526       18,740

Log-Likelihood = -14,576
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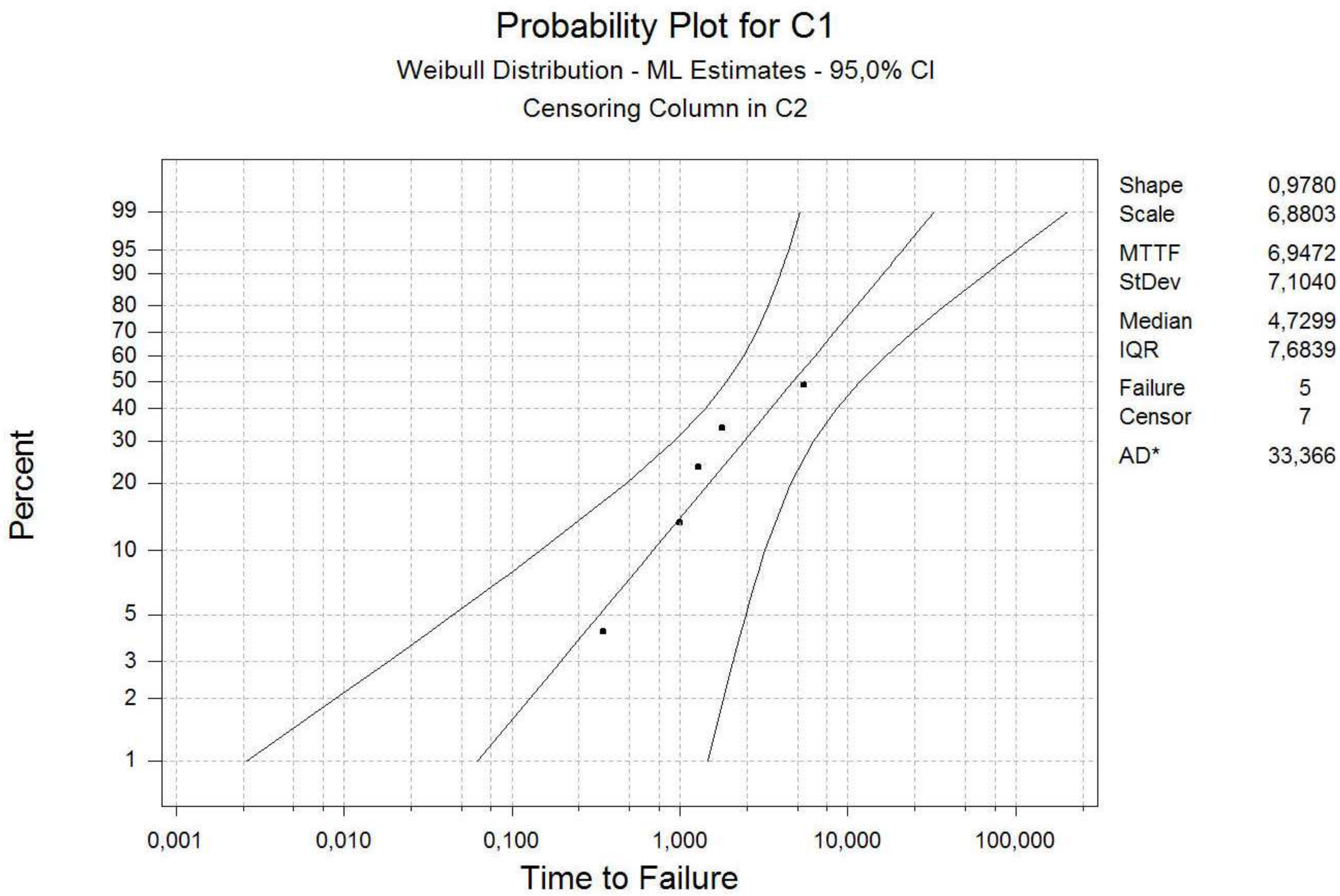
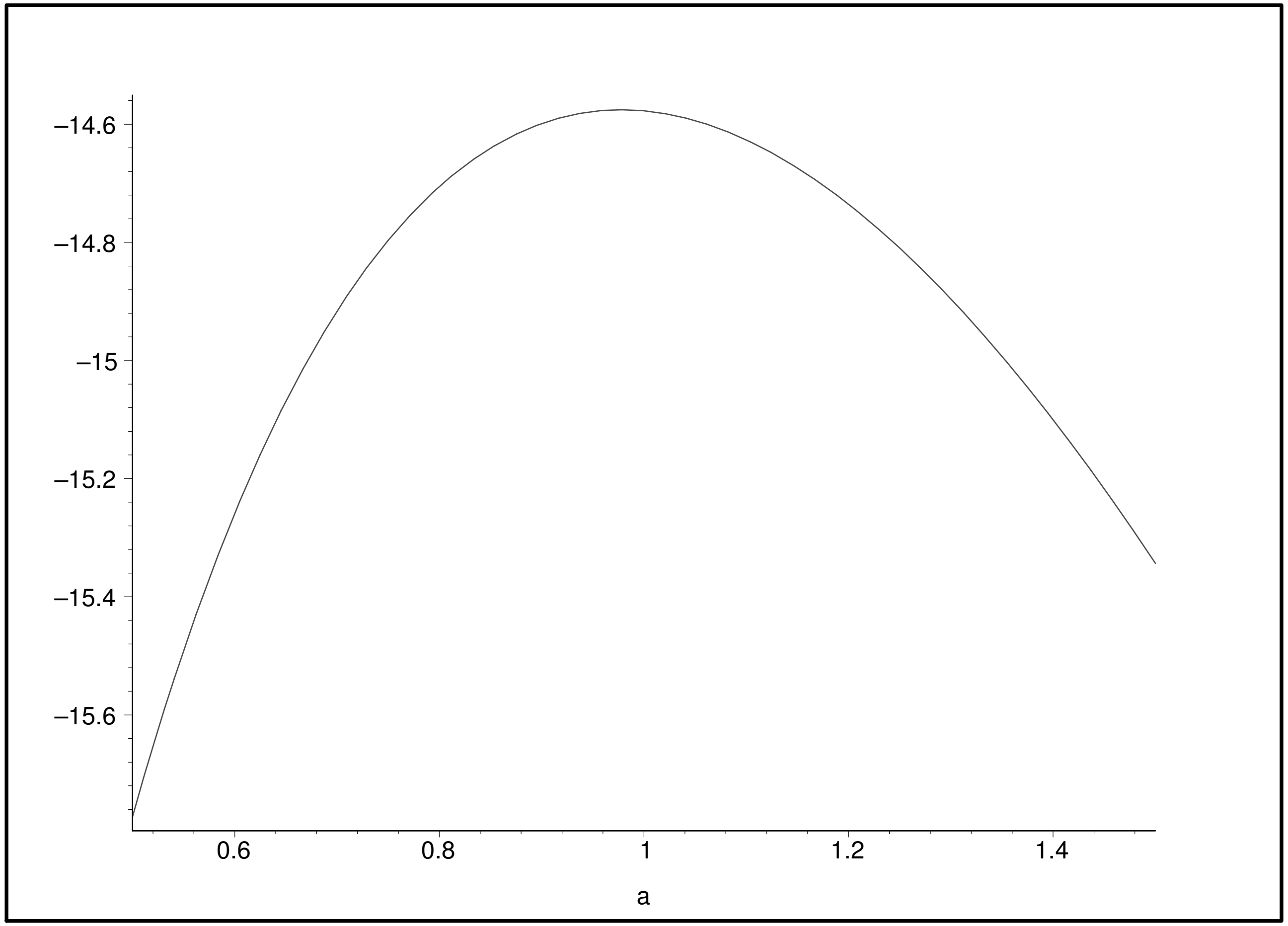
PLOT USING MINITAB 13
Plots the points

$$(\ln t_i,\ \ln(-\ln(1 - F(t_i;\theta))))$$



PROFILE LIKELIHOOD FOR SHAPE PARAMETER α IN WEIBULL DISTRIBUTION



BASED ON OUTPUT FROM MINITAB-MACRO "barlowpro"

Row	Time	Censor	No at risk	Total time	Cum total time	
1	0,35	1	12	4,20	4,20	4,2
2	0,50	0	11	1,65	5,85	10,6
3	0,75	0	10	2,50	8,35	13,0
4	1,00	1	9	2,25	10,60	16,5
5	1,30	1	8	2,40	13,00	33,2
6	1,80	1	7	3,50	16,50	
7	3,00	0	6	7,20	23,70	
8	3,15	0	5	0,75	24,45	
9	4,85	0	4	6,80	31,25	
10	5,50	1	3	1,95	33,20	
11	5,50	0	2	0,00	33,20	
12	6,25	0	1	0,75	33,95	

Row	i/n	TTT
1	0,2	0,12651
2	0,4	0,31928
3	0,6	0,39157
4	0,8	0,49699
5	1,0	1,00000

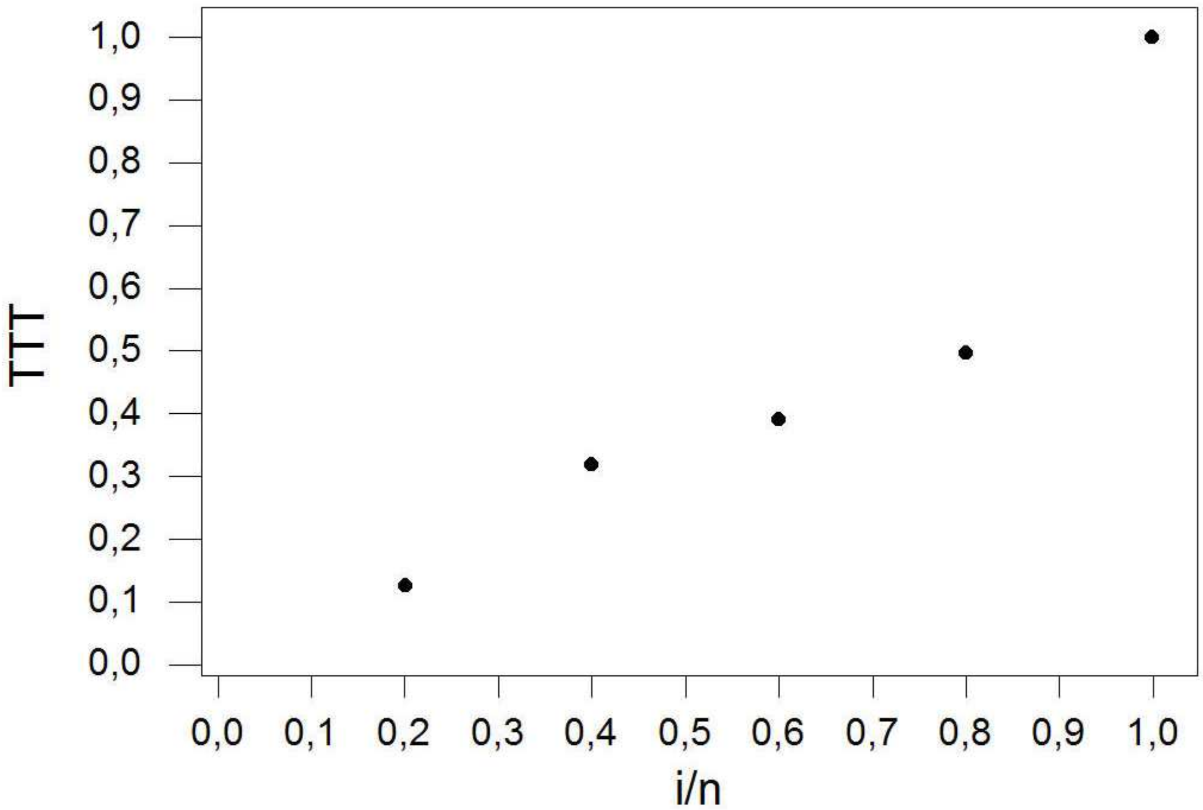
BARLOW-PROSCHAN'S TEST:

W = (4.2+10.6+13.0+16.5)/33.2 = 1.33

alt. W = 0.13 + 0.32 + 0.39 + 0.50 = 1.34

(k-1 = 4)

TTT-plot censored data



Method – Probability Plot for Uncensored/Right Censored Data

main topic

Probability plots are based on a scheme that plots the observed failure times (or a transformation of the failure times) on the x-axis vs. the estimated cumulative probabilities (p) on the y-axis. The cumulative probabilities are transformed. Transformations of both the x and y data are needed to ensure that the plotted y values are a linear function of the plotted x values. To help assess the linearity of the plotted data, a fitted line is also drawn on the probability plots. The table below shows how the actual x and y plot points are constructed:

Distribution	x coordinate	y coordinate
Weibull	ln(failure time)	ln(-ln(1 - p))
Extreme value	failure time	ln(-ln(1 - p))
Exponential	failure time	$-\ln(1-p)$
Normal	failure time	$\Phi^{-1}(p)$
Lognormal basee	ln(failure time)	$\Phi^{-1}(p)$
Lognormal base10	log10(failure time)	$\Phi^{-1}(p)$
Logistic	failure time	$\ln\left(\frac{p}{1-p}\right)$
Loglogistic	ln(failure time)	$\ln\left(\frac{p}{1-p}\right)$

where

$\Phi^{-1}(p)$ = value of standard normal distribution, Z, such that prob $(Z <= \Phi^{-1}(p) = p)$

ln (x) = natural log of x

log10(x) = log base 10 of x

Minitab estimates the cumulative probabilities (p) using the Default, Modified Kaplan-Meier, Herd-Johnson, or Kaplan-Meier method. The Default method is the normal score for uncensored data; the modified Kaplan-Meier method for censored data. Each of these methods generates nonparametric estimates of F(t), the cumulative distribution function for the random variable T, which is time to failure.

Note If the largest observation is uncensored, the Kaplan-Meier method results in p=1 for the largest uncensored observation. In this case, the Kaplan-Meier estimate for the largest observation results in a number that cannot be used in the plot. This problem is corrected by recalculating the largest p as 90% of the distance between the prior p and 1.

This table displays the differences among the methods. For a sample of n observations, let x(1), x(2),...,x(n) be the order statistics, or the data ordered from smallest to largest. Then i is the rank of the ith ordered observation x(i). The formulas are as follows:

For uncensored data...

This method . . .	Uses this equation
Normal score (default)	$\frac{i - 3/8}{n + 1/4}$
Modified Kaplan-Meier	$\frac{i - 1/2}{n}$
Herd-Johnson	$\frac{i}{n + 1}$
Kaplan-Meier	$\frac{i}{n}$

For censored data...

This method...	Uses this equation
Modified Kaplan-Meier (default)	$p_i = 1 - \frac{(1 - p'_i) + (1 - p'_{i-1})}{2}$ where p'_i is the p_i from the Kaplan-Meier estimate and $p'_0 = 0$.
Herd-Johnson estimate	$p_i = 1 - \prod_{j=1:i} \left(\frac{(n - j + 1)}{(n - j + 2)}\right)^{\delta_j}$
Kaplan-Meier product limit estimate	$p_i = 1 - \prod_{j=1:i} \left(\frac{(n - j)}{(n - j + 1)}\right)^{\delta_j}$

where

δj = 0 if the jth observation is censored

1 if the jth observation is uncensored

If there are tied failure times in the data, either all points (default), the average (median), or the maximum of the tied points is plotted. If the tie involves failures and suspensions, the failures are considered to occur before the suspensions.