FORELESNING 7

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TMA4275 LEVETIDSANALYSE

Bo Lindqvist

Institutt for matematiske fag

NTNU

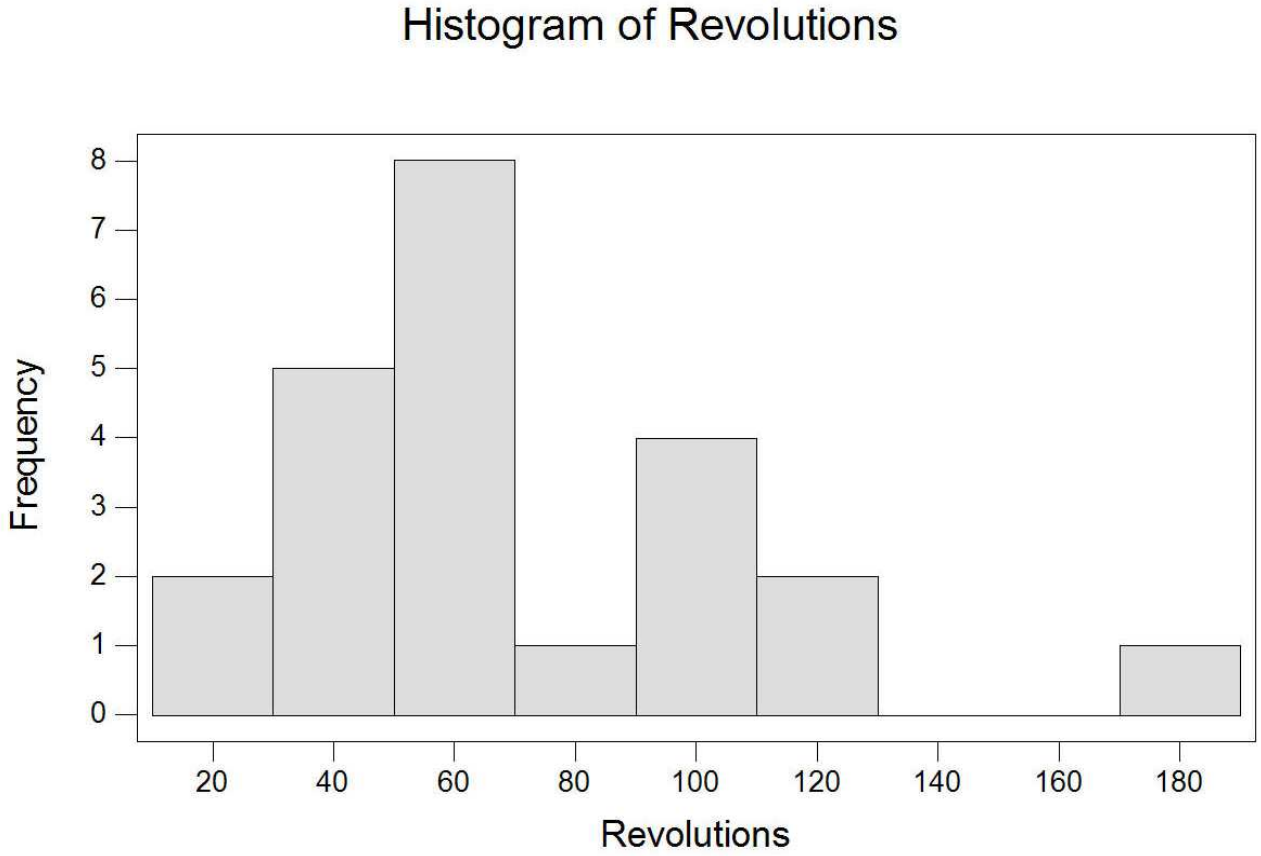
bo@math.ntnu.no

http://www.math.ntnu.no/∼bo/

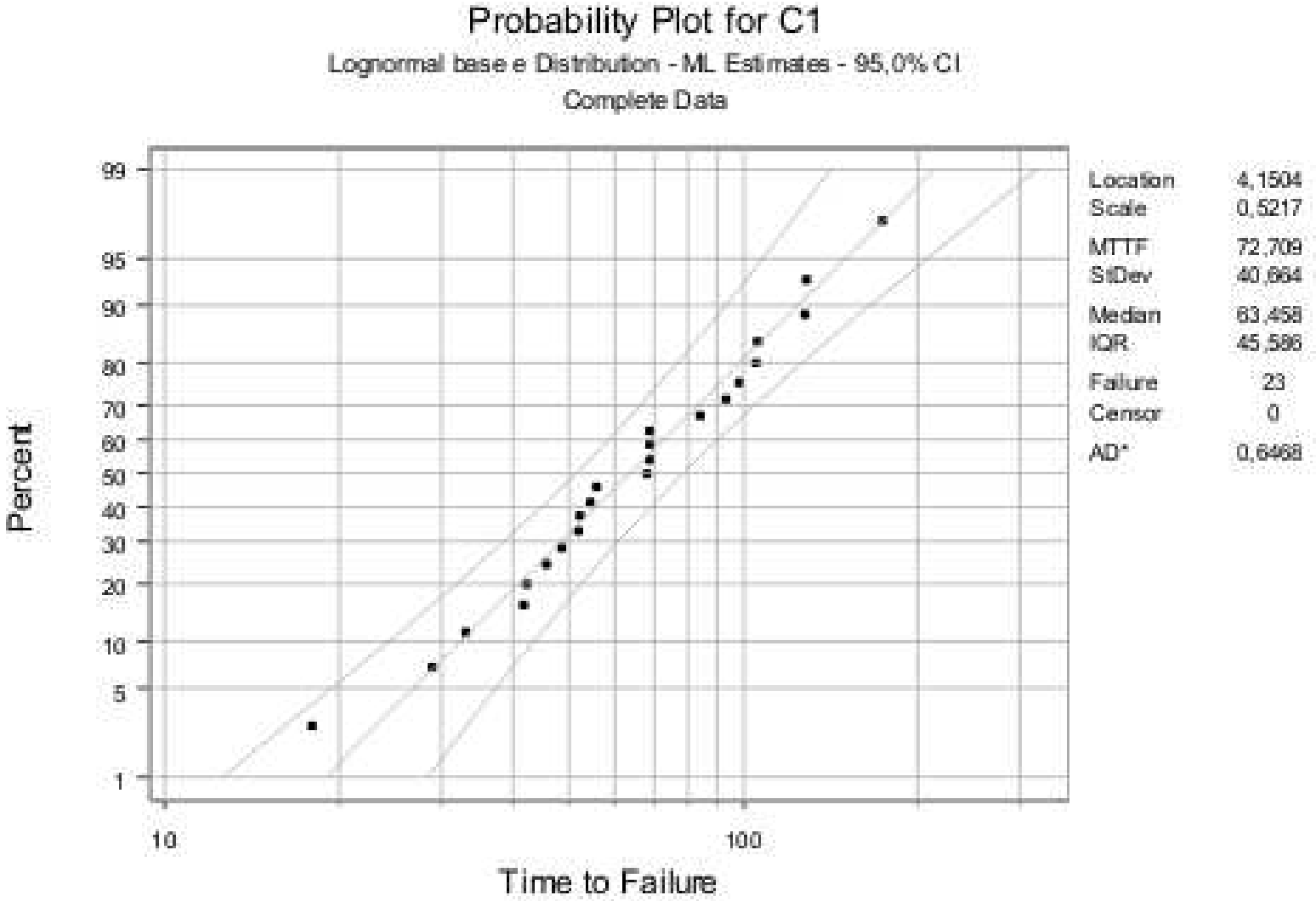
BALL BEARINGS FAILURE DATA

Data: Millions of revolutions to fatigue failure for 23 units (No censoring)

17,88	28,92	33,00	41,52	42,12	45,60	48,40	51,84
51,96	54,12	55,56	67,80	68,64	68,64	68,88	84,12
93,12	98,64	105,12	105,84	127,92	128,04	173,40	



Ball Bearings Data: Lognormal distribution



Distribution Analysis: C1

Variable: C1

Censoring Information	Count
Uncensored value	23

Estimation Method: Maximum Likelihood
Distribution: Lognormal base e

Parameter Estimates

Parameter	Estimate	Standard Error	95,0% Normal CI Lower	Upper
Location	4,1504	0,1088	3,9372	4,3636
Scale	0,52169	0,07692	0,39076	0,69649

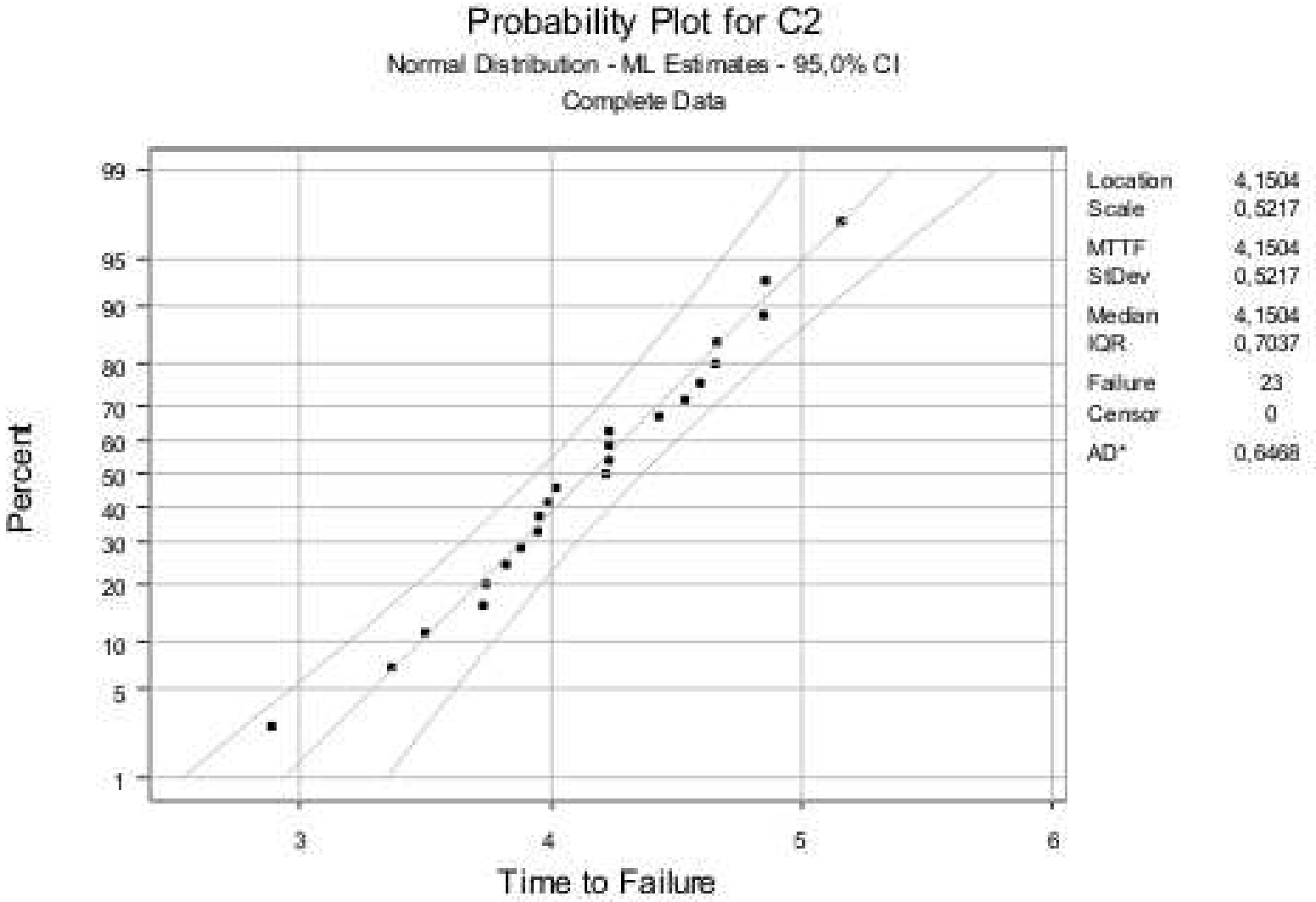
Log-Likelihood = -113,129

Goodness-of-Fit
Anderson-Darling (adjusted) = 0,6468

Characteristics of Distribution

	Estimate	Standard Error	95,0% Normal CI Lower	Upper
Mean(MTTF)	72,7087	8,4302	57,9288	91,2596
Standard Deviation	40,6644	9,5646	25,6452	64,4798
Median	63,4583	6,9029	51,2738	78,5383
First Quartile(Q1)	44,6347	5,3793	35,2442	56,5271
Third Quartile(Q3)	90,2203	10,8731	71,2393	114,2587
Interquartile Range(IQR)	45,5857	8,5755	31,5284	65,9105

Taking Log of Ball Bearings Data and fit to Normal distribution



Variable: C2

Censoring Information	Count
Uncensored value	23

Estimation Method: Maximum Likelihood
Distribution: Normal

Parameter Estimates

Parameter	Estimate	Standard Error	95,0% Normal CI Lower	Upper
Location	4,1504	0,1088	3,9372	4,3636
Scale	0,52169	0,07692	0,39076	0,69649

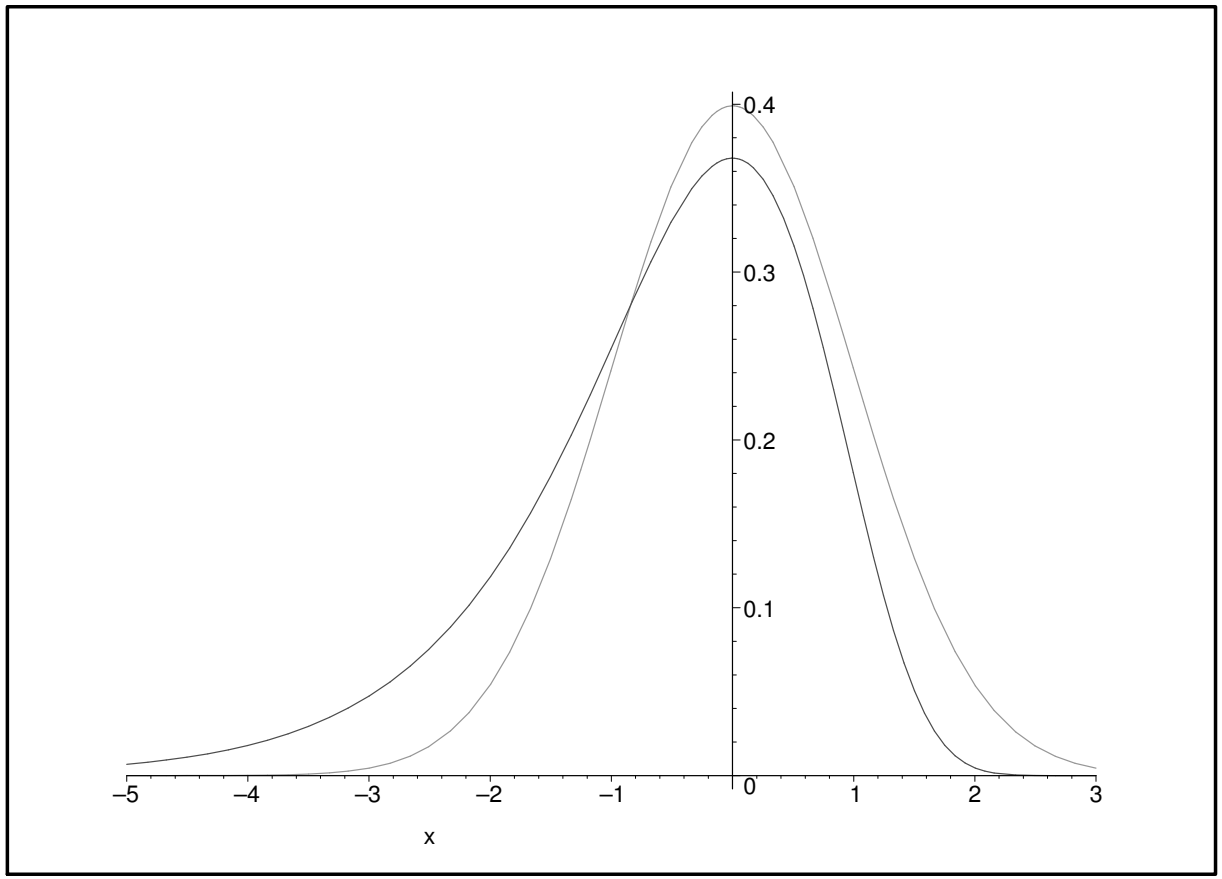
Log-Likelihood = -17,670

Goodness-of-Fit
Anderson-Darling (adjusted) = 0,6468

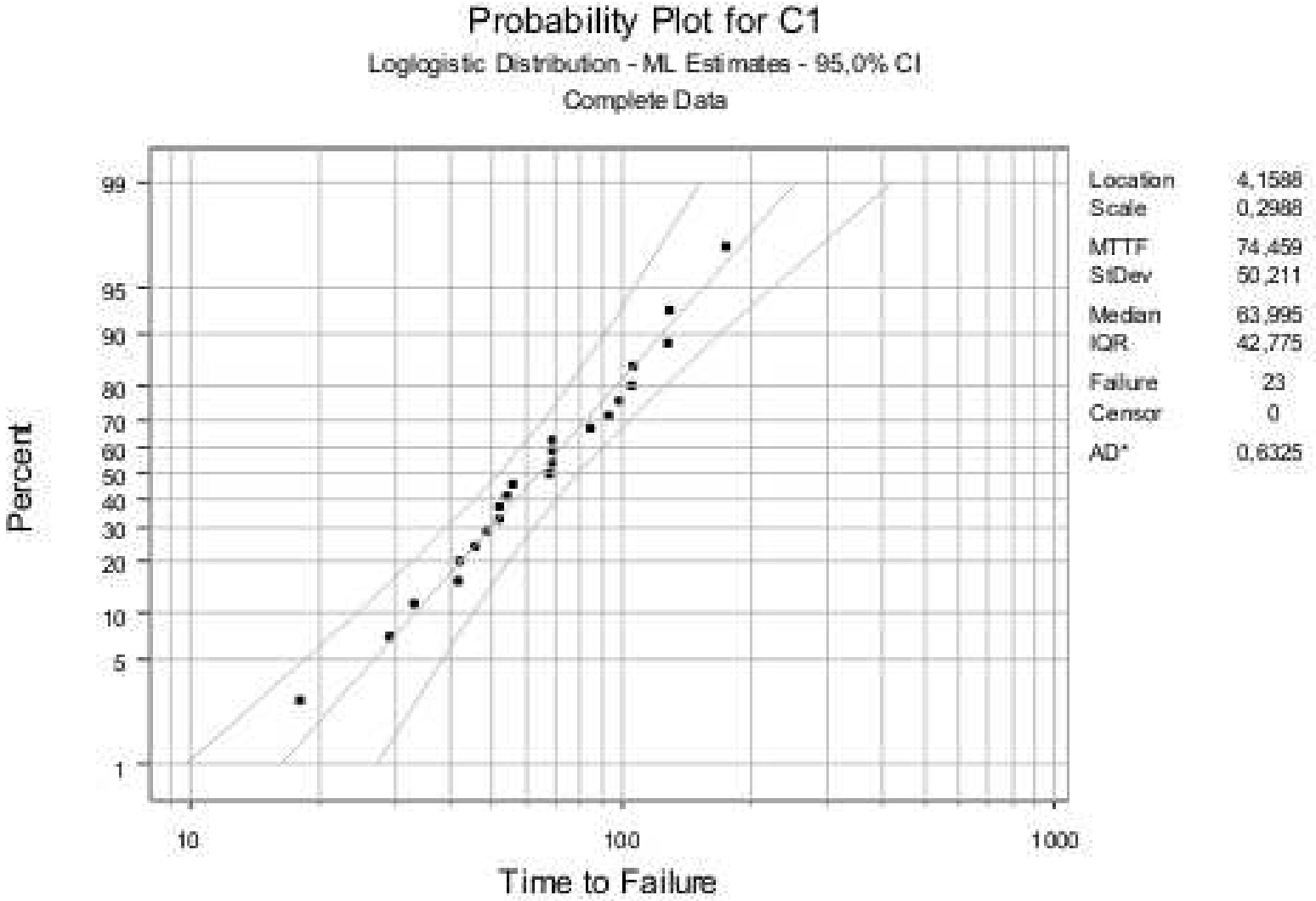
Characteristics of Distribution

	Estimate	Standard Error	95,0% Normal CI Lower	Upper
Mean(MTTF)	4,1504	0,1088	3,9372	4,3636
Standard Deviation	0,5217	0,07692	0,3908	0,6965
Median	4,1504	0,1088	3,9372	4,3636
First Quartile(Q1)	3,7985	0,1205	3,5623	4,0347
Third Quartile(Q3)	4,5023	0,1205	4,2660	4,7385
Interquartile Range(IQR)	0,7037	0,1038	0,5271	0,9395

GUMBEL-FORDELINGEN OG NORMALFORDELINGEN MED $\mu = 0, \sigma = 1$



Ball Bearings Data: Loglogistic distribution



Variable: C1

Censoring Information	Count
Uncensored value	23

Estimation Method: Maximum Likelihood
Distribution: Loglogistic

Parameter Estimates

Parameter	Estimate	Standard Error	95,0% Normal CI Lower	Upper
Location	4,1588	0,1090	3,9451	4,3725
Scale	0,29881	0,05153	0,21312	0,41897

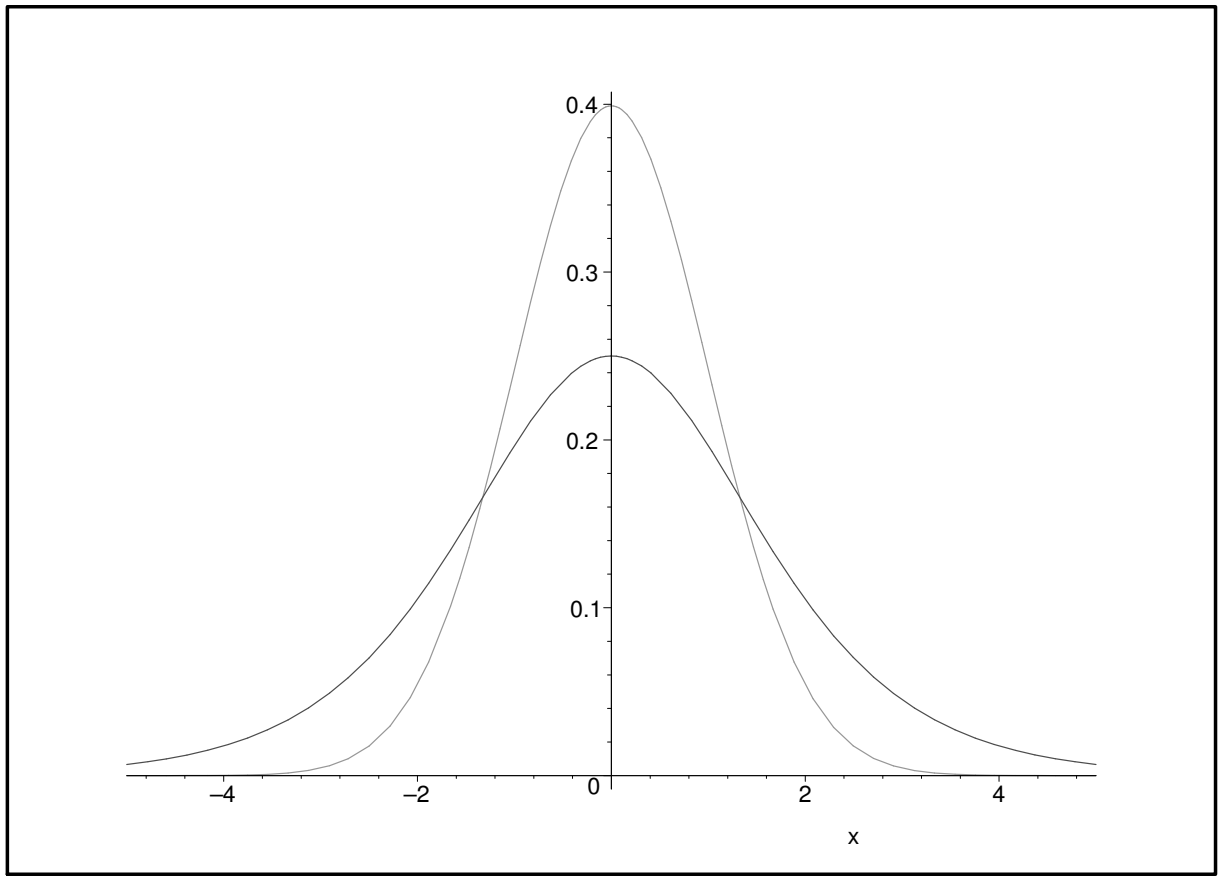
Log-Likelihood = -113,373

Goodness-of-Fit
Anderson-Darling (adjusted) = 0,6325

Characteristics of Distribution

	Estimate	Standard Error	95,0% Normal CI Lower	Upper
Mean(MTTF)	74,4589	9,0701	58,6447	94,5377
Standard Deviation	50,2113	17,0541	25,8044	97,7034
Median	63,9947	6,9784	51,6801	79,2438
First Quartile(Q1)	46,0866	5,6542	36,2363	58,6144
Third Quartile(Q3)	88,8616	10,9338	69,8199	113,0963
Interquartile Range(IQR)	42,7750	8,9647	28,3658	64,5036

LOGISTISK FORDELING OG NORMALFORDELINGEN MED $\mu = 0, \sigma = 1$



Confidence Interval for the Mean Life of a New Insulating Material

- A life test for a new insulating material used 25 specimens which were tested simultaneously at a high voltage of 30 kV.
- The test was run until 15 of the specimens failed.
- The 15 failure times (hours) were recorded as:

  1.08, 12.20, 17.80, 19.10, 26.00, 27.90, 28.20, 32.20, 35.90, 43.50, 44.00, 45.20, 45.70, 46.30, 47.80

  Then $TTT = 1.08 + \cdots + 47.80 + 10 \times 47.80 = 950.88$ hours.

- The ML estimate of θ and a 95% confidence interval are:

$$\begin{aligned} \widehat{\theta} &= 950.88/15 = 63.392 \text{ hours} \\ \left[\underline{\theta}, \tilde{\theta}\right] &= \left[\frac{2(950.88)}{\chi^2_{(.975;30)}}, \frac{2(950.88)}{\chi^2_{(.025;30)}}\right] = \left[\frac{1901.76}{46.98}, \frac{1901.76}{16.79}\right] \\ &= [40.48, \quad 113.26]. \end{aligned}$$

Pike (1966) cancer data for rats

Row	Y	D
1	143	1
2	164	1
3	188	1
4	188	1
5	190	1
6	192	1
7	206	1
8	209	1
9	213	1
10	216	1
11	220	1
12	227	1
13	230	1
14	234	1
15	246	1
16	265	1
17	304	1
18	216	0
19	244	0

Pike (1966) cancer data for rats: 3-parameter Weibull

Distribution Analysis: C1

Variable: C1

Censoring Information	Count
Uncensored value	17
Right censored value	2

Censoring value: C2 = 0

Estimation Method: Maximum Likelihood

Distribution: 3-Parameter Weibull

Parameter Estimates

Parameter	Estimate	Standard Error	95,0% Normal CI Lower	Upper
Shape	2,71148	1,05876	1,26135	5,82878
Scale	108,383	32,5734	60,1367	195,335
Threshold	122,026	28,6924	65,7898	178,262

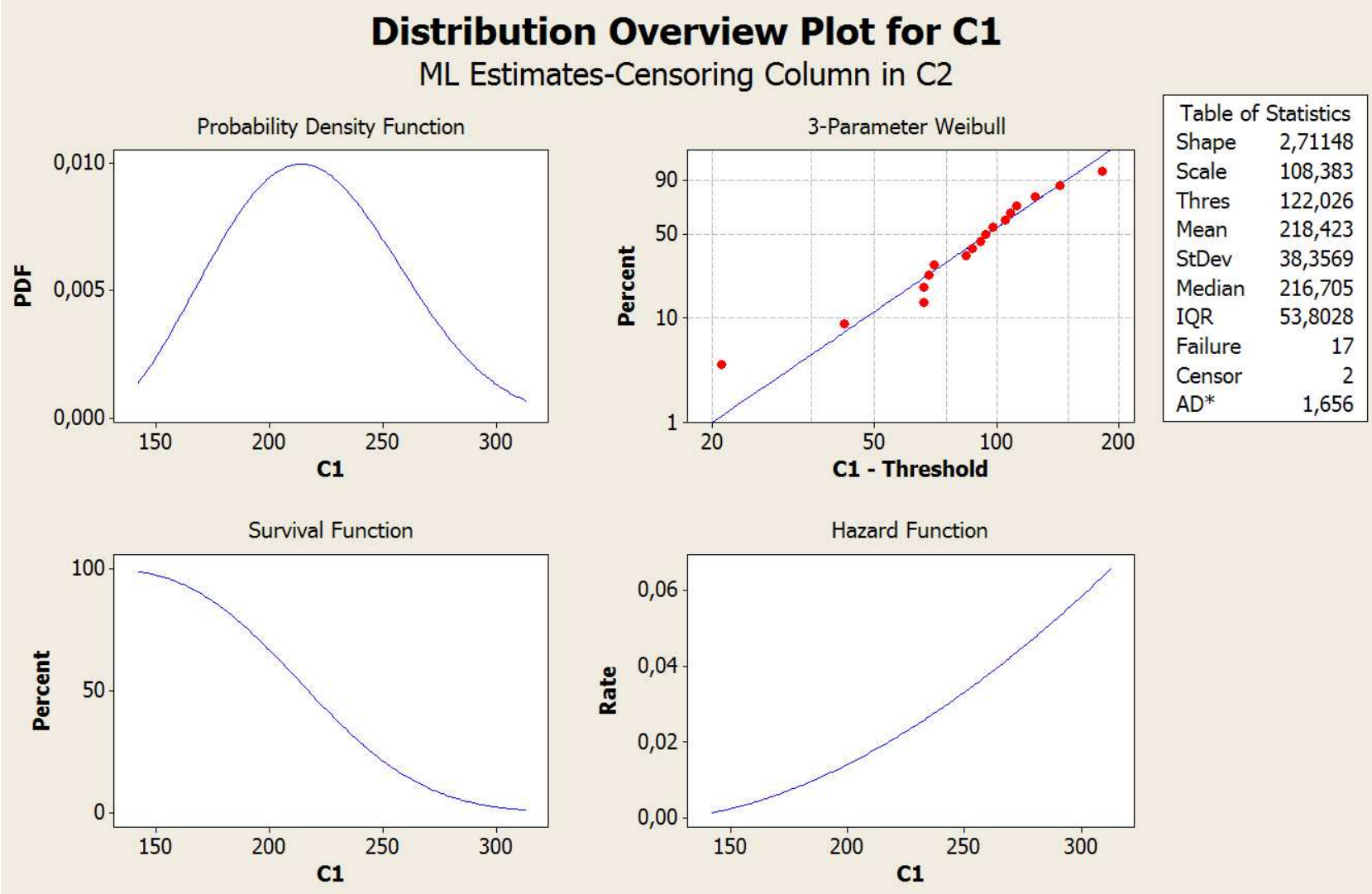
Log-Likelihood = -87,324

Goodness-of-Fit
Anderson-Darling (adjusted) = 1,656

Characteristics of Distribution

	Estimate	Standard Error	95,0% Normal CI Lower	Upper
Mean(MTTF)	218,423	8,99156	201,492	236,777
Standard Deviation	38,3569	6,41597	27,6352	53,2383
Median	216,705	9,89384	198,156	236,991
First Quartile(Q1)	190,481	9,63934	172,495	210,342
Third Quartile(Q3)	244,284	11,0118	223,627	266,849
Interquartile Range(IQR)	53,8028	8,97770	38,7945	74,6172

Pike (1966) cancer data for rats
3-parameter Weibull



Pike 3-parameter Weibull: Profile likelihood for γ

Table 4.4. m.l.e.'s and Profile Relative Likelihood for γ

γ	$\hat{\alpha}(\gamma)$	$\hat{\beta}(\gamma)$	$\ell_p(\gamma)$	$R_p(\gamma)$	$\Lambda(\gamma)$
0	234.3	6.08	−88.233	.403	1.818
60	173.2	4.49	−87.831	.602	1.015
100	131.8	3.38	−87.467	.867	.285
110	121.2	3.08	−87.381	.945	.113
120	110.6	2.78	−87.327	.998	.004
122	108.4	2.71	−87.324	1.000	.000
125	105.2	2.61	−87.330	.994	.012
130	99.7	2.44	−87.382	.944	.115
135	94.0	2.24	−87.542	.804	.436
140	88.0	1.99	−88.064	.477	1.480
142	85.2	1.80	−88.773	.235	2.896
143	81.1	1.00	−91.718	.012	8.846

Loglikelihood for truncated exponential data