LECTURE WEEK 10

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TMA4275 LIFETIME ANALYSIS

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Insulate.MTW ***

↓	C1	C2	C3	C4	C5-T	C6	C7	C8	C9	C10	C11
	Temp	ArrTemp	Plant	FailureT	Censor	Design	NewTemp	ArrNewT	NewPlant		
1	170	26,1865	1	343	F	80	80	32,8600	1		
2	170	26,1865	1	869	F	100	80	32,8600	2		
3	170	26,1865	1	244	C		100	31,0988	1		
4	170	26,1865	1	716	F		100	31,0988	2		
5	170	26,1865	1	531	F						
6	170	26,1865	1	738	F						
7	170	26,1865	1	461	F						
8	170	26,1865	1	221	F						
9	170	26,1865	1	665	F						
10	170	26,1865	1	384	C						
11	170	26,1865	2	394	C						
12	170	26,1865	2	369	F						
13	170	26,1865	2	366	F						
14	170	26,1865	2	507	F						
15	170	26,1865	2	461	F						
16	170	26,1865	2	431	F						
17	170	26,1865	2	479	F						
18	170	26,1865	2	106	F						
19	170	26,1865	2	545	F						
20	170	26,1865	2	536	F						
21	150	27,4242	1	2134	C						
22	150	27,4242	1	2746	F						
23	150	27,4242	1	2859	F						
24	150	27,4242	1	1826	C						

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Example of Accelerated Life Testing

main topic   interpreting results   session command   see also

Suppose you want to investigate the deterioration of an insulation used for electric motors. The motors normally run between 80 and 100° C. To save time and money, you decide to use accelerated life testing.

First you gather failure times for the insulation at abnormally high temperatures – 110, 130, 150, and 170° C – to speed up the deterioration. With failure time information at these temperatures, you can then extrapolate to 80 and 100° C. It is known that an Arrhenius relationship exists between temperature and failure time.

To see how well the model fits, you will draw a probability plot based on the standardized residuals.

1 Open the worksheet INSULATE.MTW.
2 Choose **Stat > Reliability/Survival > Accelerated Life Testing**.
3 In **Variables/Start variables**, enter **FailureT**. In **Accelerating variable**, enter **Temp**.
4 From **Relationship**, choose **Arrhenius**.
5 Click **Censor**. In **Use censoring columns**, enter **Censor**, then click **OK**.
6 Click **Graphs**. In **Enter design value to include on plot**, enter **80**. Click **OK**.
7 Click **Estimate**. In **Enter new predictor values**, enter **Design**, then click **OK** in each dialog box.

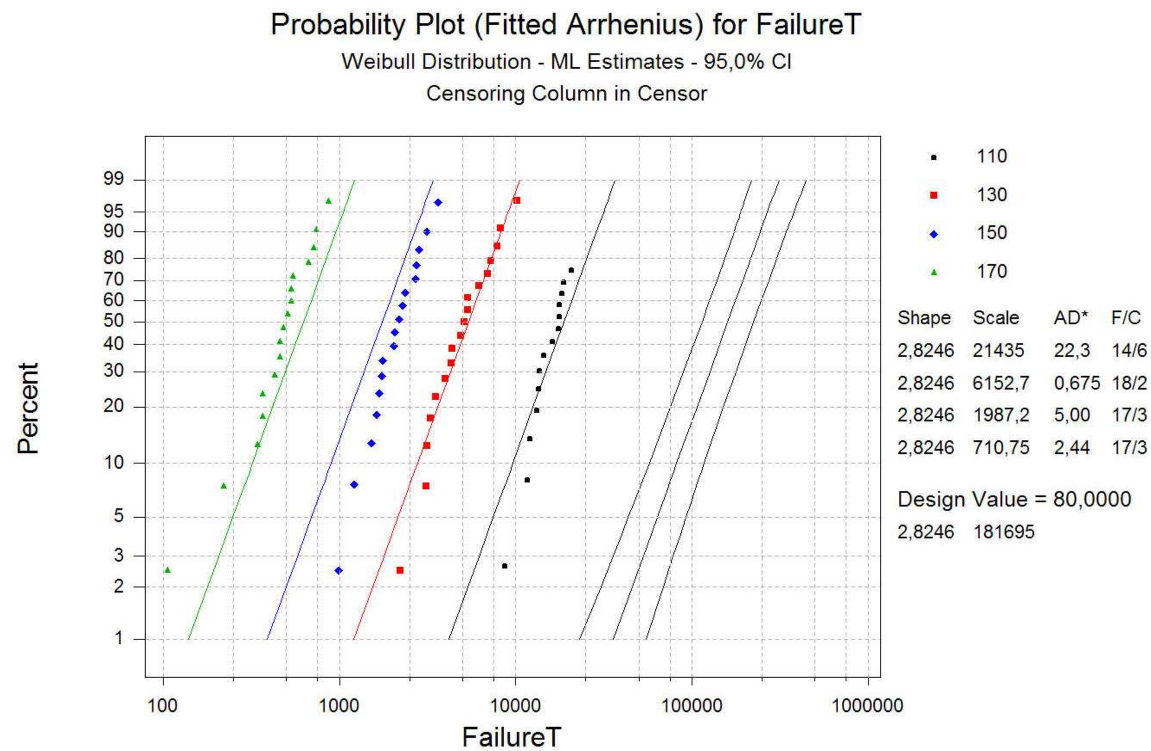
Session window output

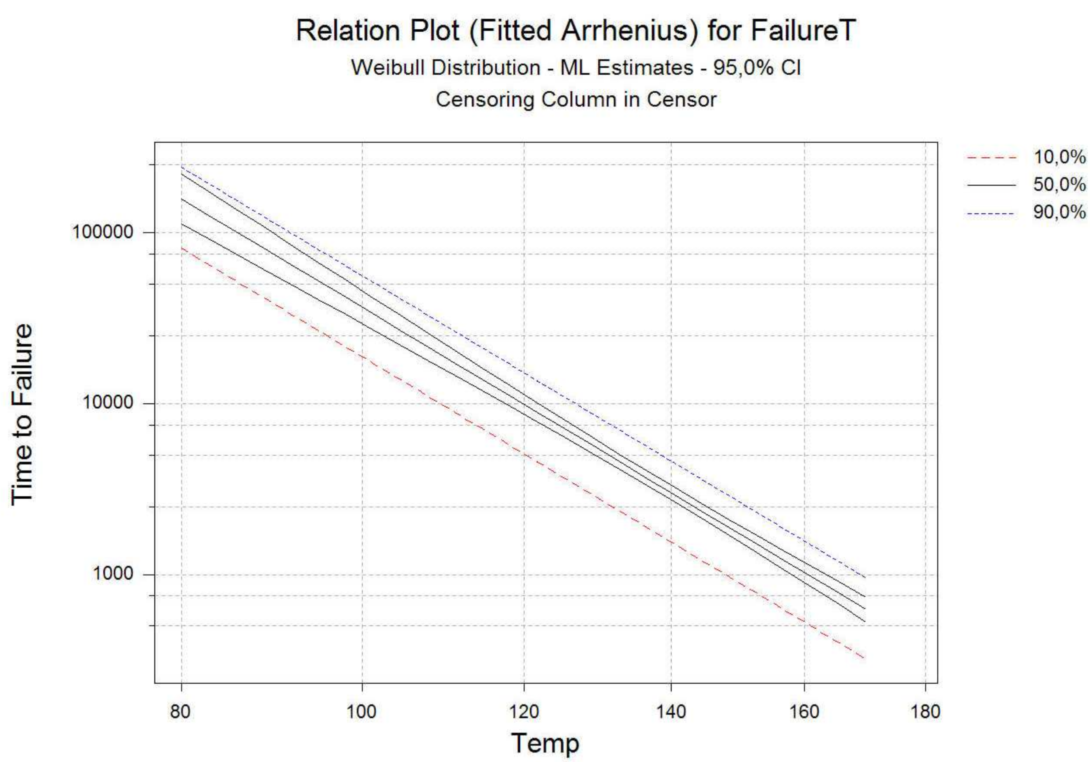
Regression with Life Data: FailureT versus Temp

```
Response Variable:  FailureT

Censoring Information              Count
Uncensored value                      66
Right censored value                  14
Censoring value:  Censor = C

Estimation Method:  Maximum Likelihood
Distribution:  Weibull
Transformation on accelerating variable:  Arrhenius
```

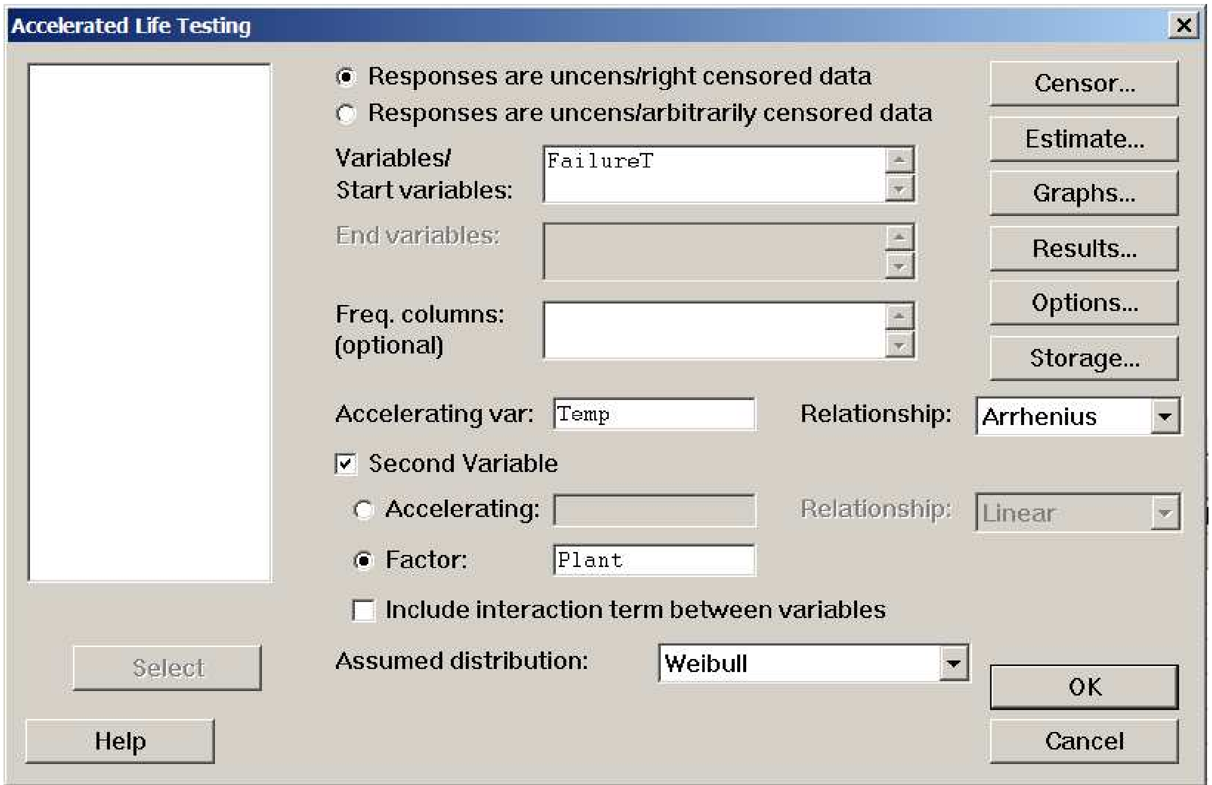


```
Regression Table
                       Standard                        95.0% Normal CI
Predictor      Coef       Error        Z      P        Lower       Upper
Intercept  -15.1874      0.9862   -15.40  0.000     -17.1203    -13.2546
Temp        0.83072     0.03504    23.71  0.000      0.76204     0.89940
Shape        2.8246      0.2570                       2.3633      3.3760

Log-Likelihood = -564.693

Anderson-Darling (adjusted) Goodness-of-Fit

At each accelerating level
Level      Fitted Model
110                   *
130                   *
150                   *
170                   *

Table of Percentiles
                                     Standard        95.0% Normal CI
 Percent        Temp  Percentile        Error       Lower       Upper
      50     80.0000    159584.5     27446.85    113918.2    223557.0
      50    100.0000    36948.57     4216.511    29543.36    46209.94
```


Probability Plot (Fitted Arrhenius) for FailureT
Weibull Distribution - ML Estimates - 95,0% CI
Censoring Column in Censor
Percent
FailureT
110
130
150
170
Shape Scale AD* F/C
2,8246 21435 22,3 14/6
2,8246 6152,7 0,675 18/2
2,8246 1987,2 5,00 17/3
2,8246 710,75 2,44 17/3
Design Value = 80,0000
2,8246 181695


Relation Plot (Fitted Arrhenius) for FailureT
Weibull Distribution - ML Estimates - 95,0% CI
Censoring Column in Censor
Time to Failure
Temp
10,0%
50,0%
90,0%

ADDING THE FACTOR "PLANT":



EXAM MAY 2003, PROBLEM 3:

Assume that a component under normal stress has survival function (reliability function) $R_0(t)$ and hazard function (failure rate) $z_0(t)$.

One wants to estimate the reliability of this component type by means of accelerated lifetime testing. This is done by measuring the lifetime (or a censored lifetime) of the component under stress s, $0 \leq s < \infty$. Normal stress corresponds to $s = 0$.

Two models are considered:

Model 1: Proportional hazards model. Under stress s the component's hazard function is

$$z_s^{PH}(t) = z_0(t)g(s)$$

for a function $g(s)$ with $g(0) = 1$.

Model 2: Accelerated lifetime model. Under stress s the component's survival function is

$$R_s^{AL}(t) = R_0(\phi(s)t)$$

for a function $\phi(s)$ with $\phi(0) = 1$.

(a) Explain briefly what is the purpose of accelerated lifetime testing. What are the ideas behind the two models? What do the functions $g(s)$ and $\phi(s)$ express?

(b) Let $R_s^{PH}(\cdot)$ be the survival function of a component under stress s in Model 1. Show that

$$R_s^{PH}(t) = R_0(t)^{g(s)}$$

Let further $z_s^{AL}(\cdot)$ be the hazard function of a component under stress s in Model 2. Express $z_s^{AL}(\cdot)$ by the functions $z_0(\cdot)$ and $\phi(\cdot)$.

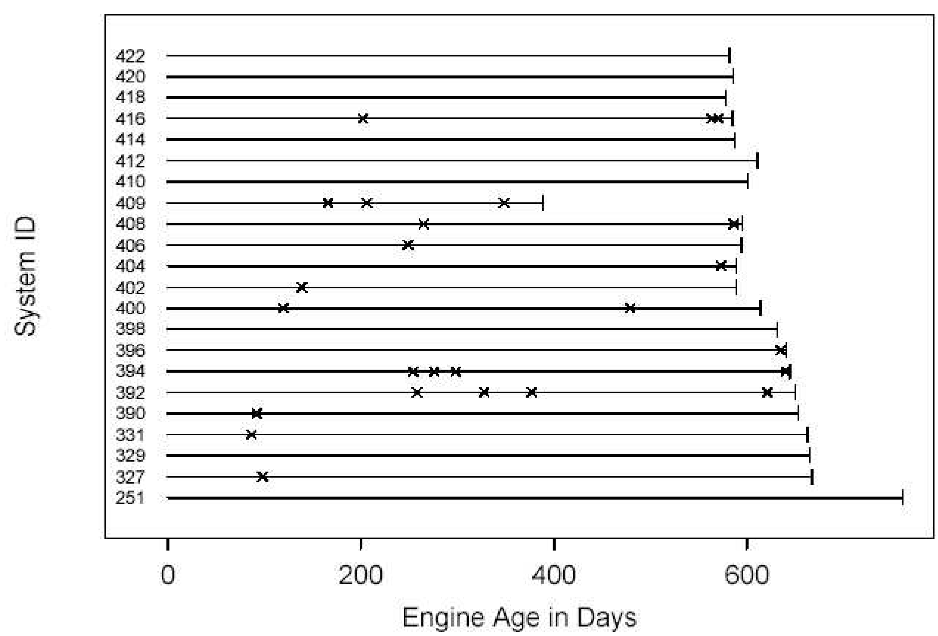
(c) Assume that the component's lifetime under normal stress is Weibull(α, θ), defined by

$$R_0(t) = e^{-(t/\theta)^\alpha}$$

Show that the lifetime under stress $s > 0$ is also Weibull-distributed under both models. What are the parameters in the corresponding Weibull-distributions?

In what sense can we say that Model 1 and Model 2 are equivalent under Weibull-distributed lifetimes?

Valve Seat Replacement Times Event Plot
(Nelson and Doganaksoy 1989)

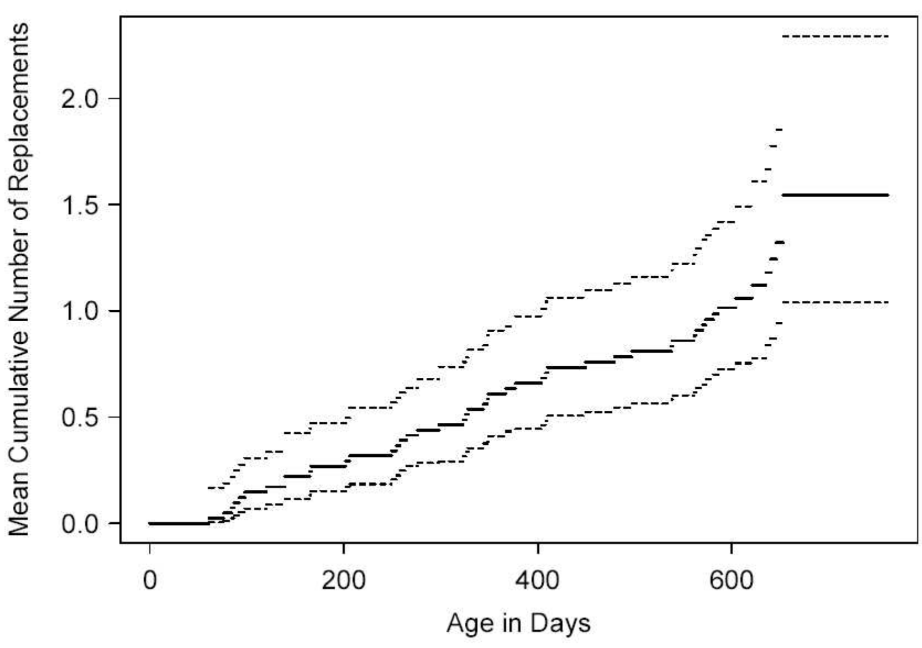


Valve Seat Replacement Times (Nelson and Doganaksoy 1989)

Data collected from valve seats from a fleet of 41 diesel engines (days of operation)

- Each engine has 16 valves
- Does the replacement rate increase with age?
- How many replacement valves will be needed in the future?
- Can valve life in these systems be modeled as a renewal process?

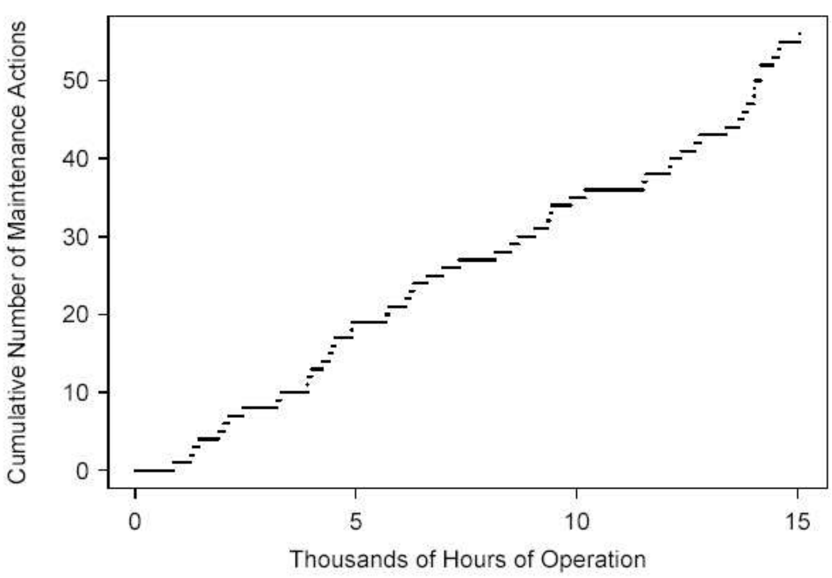
Estimate of Number of Valve Seat $\mu(t)$



Times of Unscheduled Maintenance Actions for a USS Grampus Diesel Engine

- Unscheduled maintenance actions caused by failure of imminent failure.
- Unscheduled maintenance actions are inconvenient and expensive.
- Data available for 16,000 operating hours.
- Data from Lee (1980).
- Is the system deteriorating (i.e., are failures occurring more rapidly as the system ages)?
- Can the occurrence of unscheduled maintenance actions be modeled by an HPP?

Cumulative Number of Unscheduled Maintenance Actions Versus Operating Hours for a USS Grampus Diesel Engine Lee (1980)



The Likelihood for the NHPP - Single Unit

- With **interval** recurrence data.

  Suppose that the unit has been observed for a period $(0, t_a]$ and the data are the number of recurrences $d_1, \ldots, d_m$ in the nonoverlapping intervals $(t_0, t_1]$, $(t_1, t_2]$, $\ldots$, $(t_{m-1}, t_m]$ (with $t_0 = 0$, $t_m = t_a$).

$$\begin{aligned}
L(\boldsymbol{\theta}) &= \Pr\left[N(t_0, t_1) = d_1, \ldots, N(t_{m-1}, t_m) = d_m\right] \\
&= \prod_{j=1}^{m} \Pr\left[N(t_{j-1}, t_j) = d_j\right] \\
&= \prod_{j=1}^{m} \frac{\left[\mu(t_{j-1}, t_j; \boldsymbol{\theta})\right]^{d_j}}{d_j!} \exp\left[-\mu(t_{j-1}, t_j; \boldsymbol{\theta})\right] \\
&= \prod_{j=1}^{m} \frac{\left[\mu(t_{j-1}, t_j; \boldsymbol{\theta})\right]^{d_j}}{d_j!} \times \exp\left[-\mu(t_0, t_a; \boldsymbol{\theta})\right]
\end{aligned}$$

The Likelihood for the NHPP (Continued)

- If the number of intervals m increases and there are **exact** recurrences at $t_1 \leq \ldots \leq t_r$ (here $r = \sum_{j=1}^{m} d_j$, $t_0 \leq t_1$, $t_r \leq t_a$), then using a limiting argument it follows that the likelihood in terms of the density approximation is

$$L(\boldsymbol{\theta}) = \prod_{j=1}^{r} \nu(t_j; \boldsymbol{\theta}) \times \exp\left[-\mu(0, t_a; \boldsymbol{\theta})\right]$$

- For simplicity, above we assumed that the intervals are contiguous. Obvious changes to the formula above give the likelihood when there are gaps among the intervals.

- In both cases (the interval data or exact recurrences data) the same methods used in Chapters 7, 8 can be used to obtain the ML estimate $\widehat{\boldsymbol{\theta}}$ and confidence regions for $\boldsymbol{\theta}$ or functions of $\boldsymbol{\theta}$.

CONDITIONAL ROCOF BY MINIMAL REPAIR (NHPP) AND PERFECT REPAIR (RENEWAL PROCESS)