LECTURE WEEK 12
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TMA4275 LIFETIME ANALYSIS

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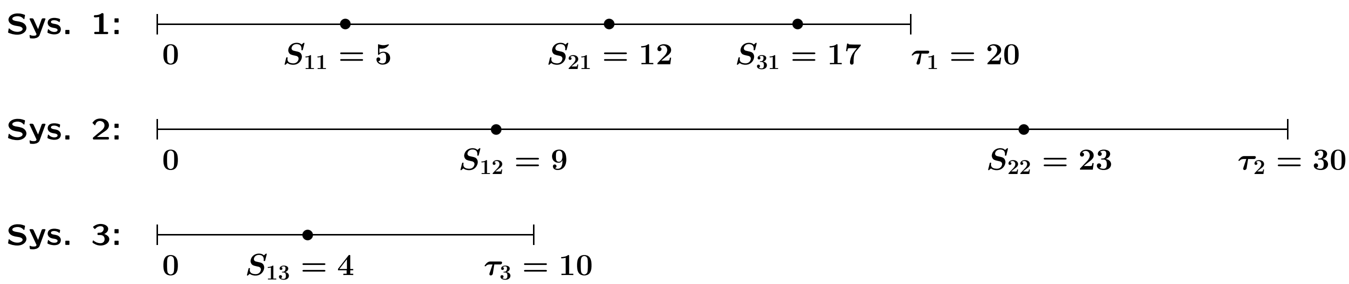
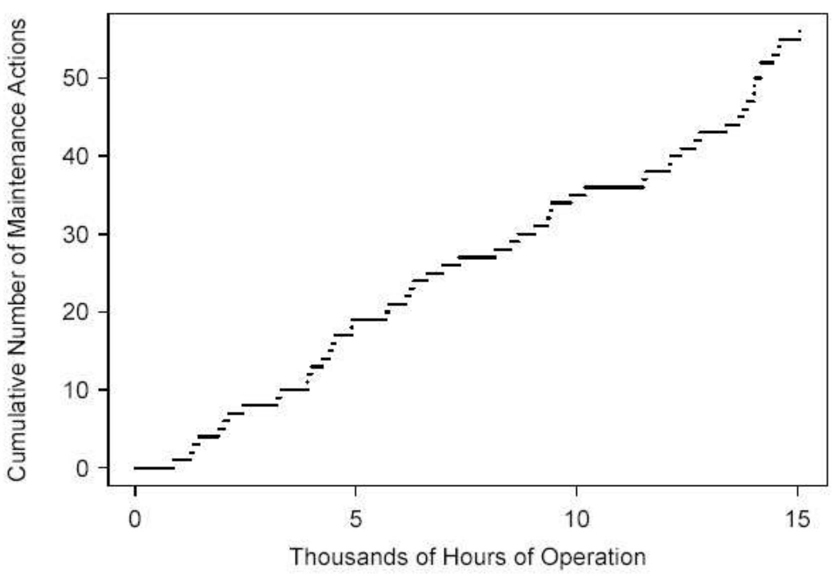
SIMPLE EXAMPLE WITH THREE SYSTEMS



Times of Unscheduled Maintenance Actions for a USS Grampus Diesel Engine

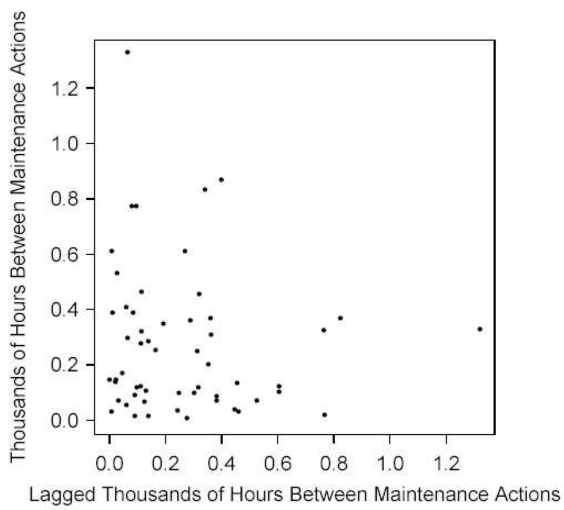
- Unscheduled maintenance actions caused by failure of imminent failure.
- Unscheduled maintenance actions are inconvenient and expensive.
- Data available for 16,000 operating hours.
- Data from Lee (1980).
- Is the system deteriorating (i.e., are failures occurring more rapidly as the system ages)?
- Can the occurrence of unscheduled maintenance actions be modeled by an HPP?

Cumulative Number of Unscheduled Maintenance Actions Versus Operating Hours for a USS Grampus Diesel Engine Lee (1980)

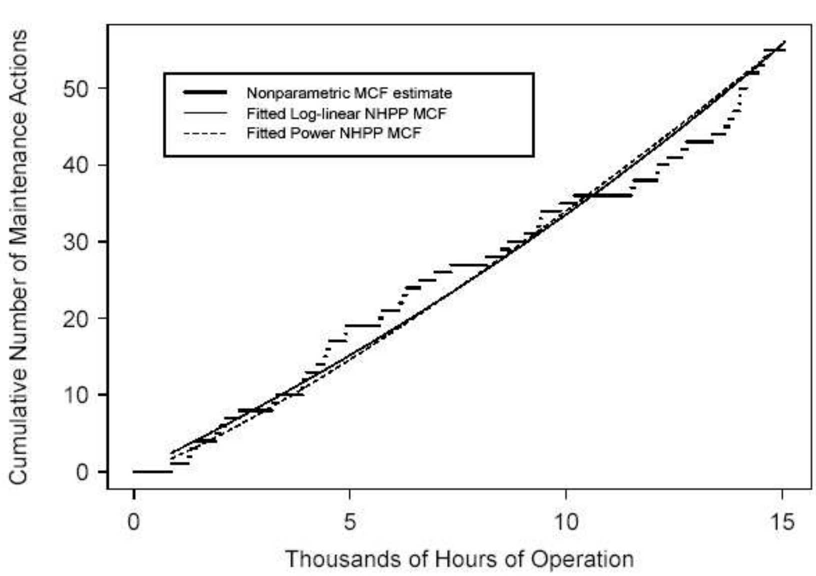


Grampus-data: Plot of (T_i, T_{i+1}) to check whether times between failures can be assumed independent. The figure does not indicate a correlation between successive interfailure times.

USS Grampus Diesel Engine
Plot of Times Between Unscheduled Maintenance Actions Versus Lagged Times Between Unscheduled Maintenance Actions



Cumulative Number of Unscheduled Maintenance Actions Versus Operating Hours with Power and Loglinear NHPP Models for a USS Grampus Diesel Engine



Results of Fitting NHPP Models to the USS Grampus Diesel Engine Data

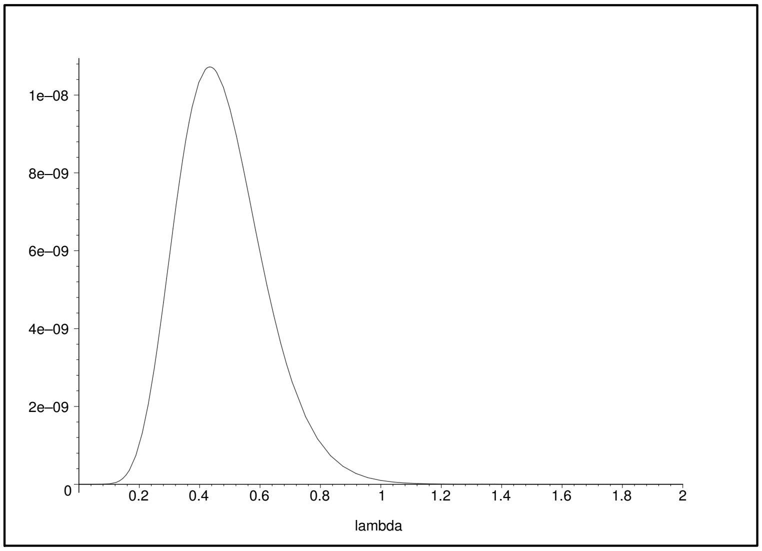
- Both models seem to fit the data very well.
- For the power recurrence rate model, $\widehat{\beta}$=1.22 and $\widehat{\eta}$ =0.553.
- For the loglinear recurrence rate model, $\widehat{\gamma}_0$=1.01 and $\widehat{\gamma}_1$ =.0377.
- Times between recurrences are consistent with a HPP:
  - the Lewis-Robinson test gave $Z_{\text{LR}} = 1.02$ with p-value $p = .21$.
  - the MIL-HDBk-189 test gave $X^2_{\text{MHB}} = 92$ with p-value $p = .08$.

Life testing of $n = 13$ airplane components (Mann and Fertig, 1976), censored after failure number $r = 10$ (Type II-censoring), resulted in:

j	Time (Y_j)	Censor
1	0,22	1
2	0,50	1
3	0,88	1
4	1,00	1
5	1,32	1
6	1,33	1
7	1,54	1
8	1,76	1
9	2,50	1
10	3,00	1
11	3,00	0
12	3,00	0
13	3,00	0

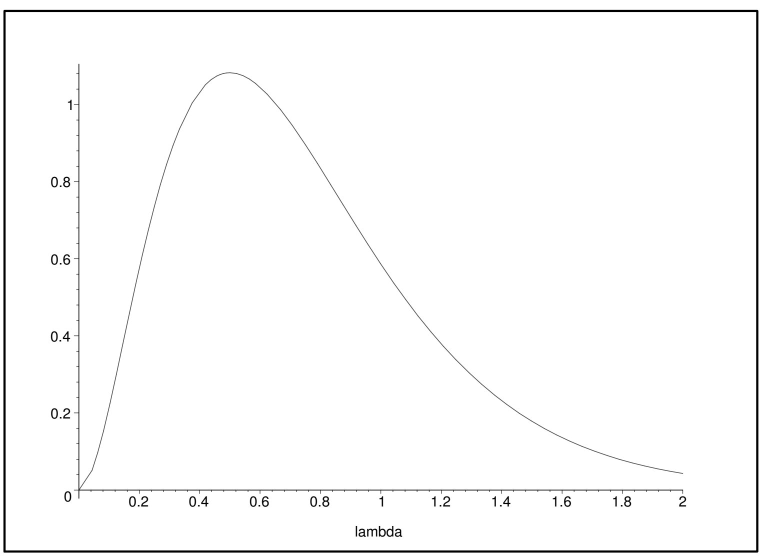
Let the model be that $T \sim$ eksp(λ), i.e. the likelihood-function is

$$L(\lambda|\text{data}) = \lambda^r e^{-\sum_{j=1}^n y_j} = \lambda^{10} e^{-23.05}$$



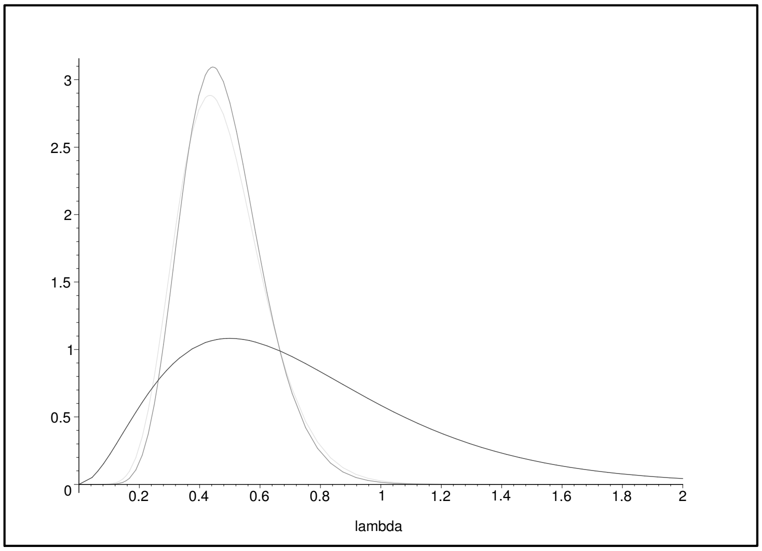
Likelihood-function for airplane component data

MLE: $\hat{\lambda} = 0.434$



Prior distribution for λ in airplane component data,
$\Lambda \sim$ Gamma$(3, 4)$

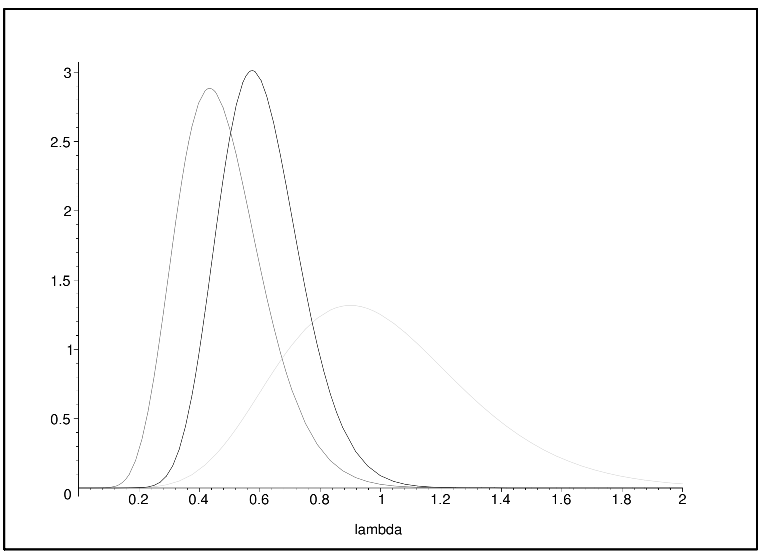
$$E(\Lambda) = 0.75,\ SD(\Lambda) = 0.43320$$



Prior distribution ("lowest"), Likelihood-function (normalized to density, "second highest") and Posterior distribution ("highest") for λ in airplane component data.

Posterior maximum is for $\lambda = 0.444$

Posterior expectation, i.e. Bayes-estimate is $\hat{\lambda}_B = 0.481$



Alternative prior distribution (Gamma(10,10), "lowest"), Likelihood function (normalized to density, "second highest") and Posterior distribution ("highest") for λ in airplane component data.

Posterior maximum is now for $\lambda = 0.575$.

Posterior expectation, i.e. Bayes-estimate is now $\hat{\lambda}_B = 0.6051$