

LECTURE WEEK 2
Spring 2005
January 18 and 21

TMA4275 LIFETIME ANALYSIS

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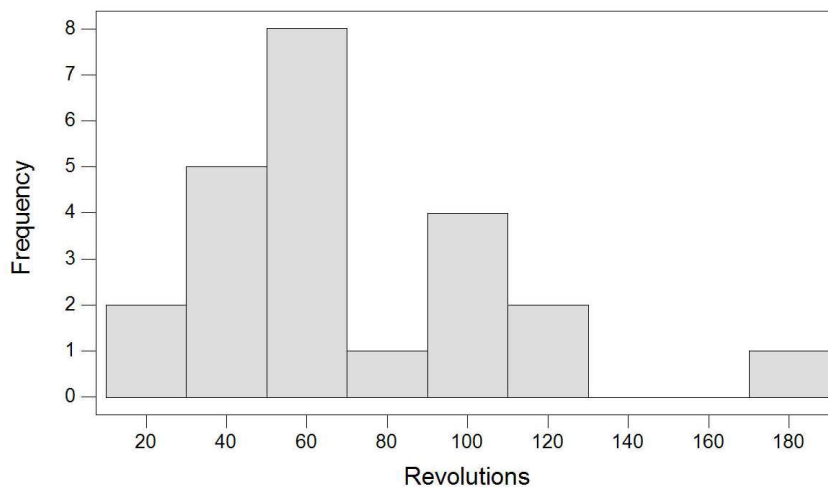
1

BALL BEARINGS FAILURE DATA

Data: Millions of revolutions until fatigue failure for 23 units

17,88	28,92	33,00	41,52	42,12	45,60	48,40	51,84
51,96	54,12	55,56	67,80	68,64	68,64	68,88	84,12
93,12	98,64	105,12	105,84	127,92	128,04	173,40	

Histogram of Revolutions

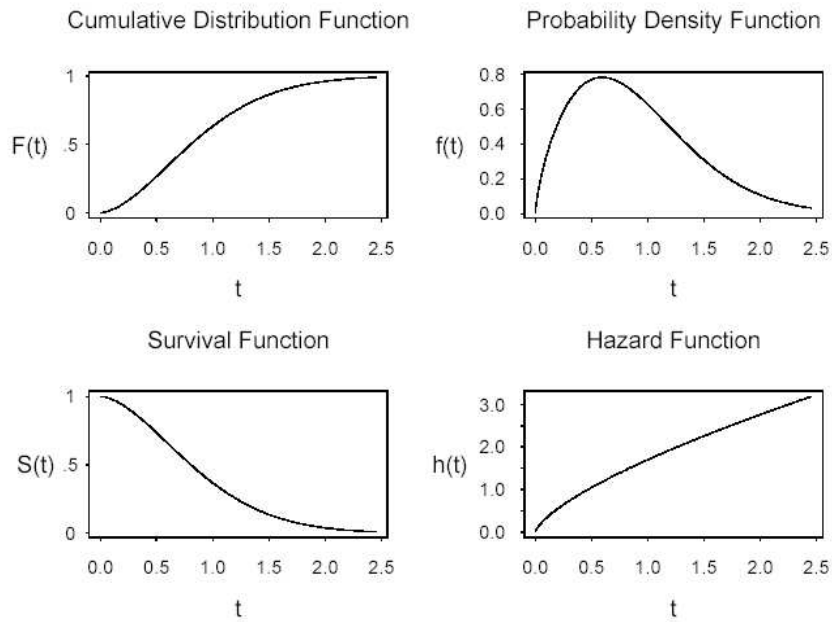


2

Typical Failure-time cdf, pdf, hf, and sf

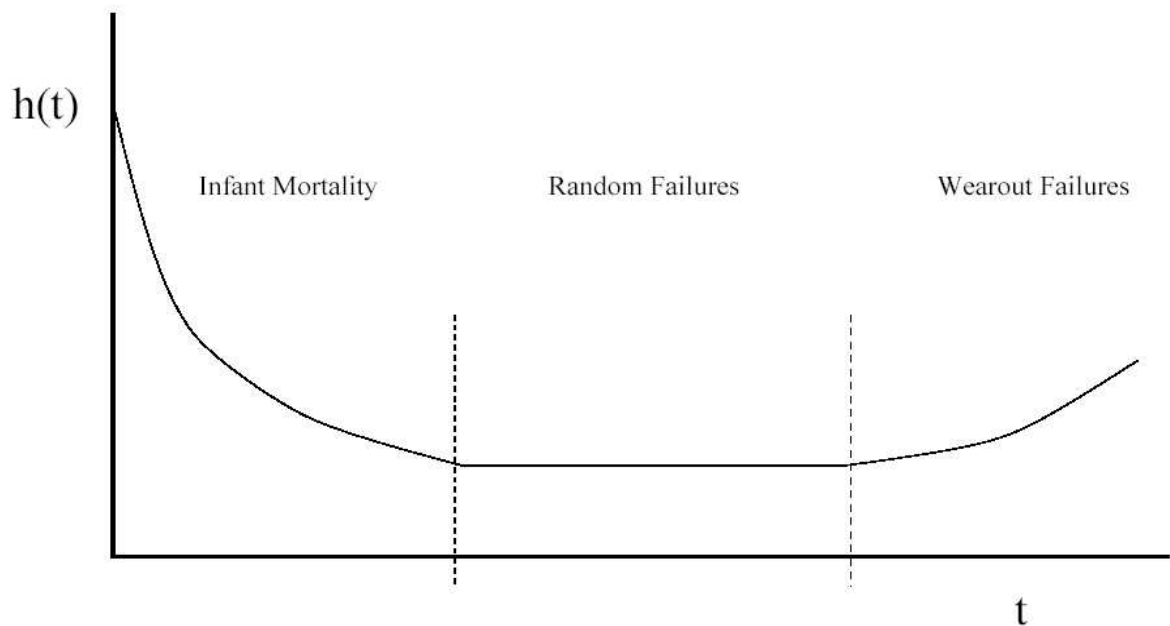
$$F(t) = 1 - \exp(-t^{1.7}); \quad f(t) = 1.7 \times t^{0.7} \times \exp(-t^{1.7})$$

$$S(t) = \exp(-t^{1.7}); \quad h(t) = 1.7 \times t^{0.7}$$



3

Bathtub Curve Hazard Function



4

10 Dødelighetstabeller 1. 2001

Alder x	Levende ved alder x		Døde i alder x til x+1		Forventet gjensleende levetid ved alder x		Dødsansynlighet for alder x, Promille, (Uglattet).	
	lx		dx		e0x		qx	
	Menn	Kvinner	Menn	Kvinner	Menn	Kvinner	Menn	Kvinner
0	100 000	100 000	424	343	76,21	81,53	4,24	3,43
1	99 576	99 657	39	27	75,53	80,81	0,39	0,28
2	99 537	99 630	33	7	74,56	79,83	0,33	0,07
3	99 504	99 623	32	17	73,58	78,84	0,33	0,17
4	99 472	99 606	13	10	72,61	77,85	0,13	0,10
5	99 459	99 596	6	10	71,62	76,86	0,06	0,10
6	99 453	99 586	22	17	70,62	75,87	0,22	0,17
7	99 431	99 569	10	3	69,64	74,88	0,10	0,03
8	99 422	99 566	9	13	68,64	73,88	0,09	0,13
9	99 412	99 552	9	3	67,65	72,89	0,09	0,03
10	99 403	99 549	12	3	66,66	71,89	0,12	0,03
11	99 390	99 546	3	10	65,66	70,89	0,03	0,10
12	99 387	99 536	16	3	64,67	69,90	0,16	0,03
13	99 371	99 532	10	11	63,68	68,90	0,10	0,11
14	99 361	99 522	7	7	62,68	67,91	0,07	0,07
15	99 354	99 514	32	11	61,69	66,92	0,32	0,11
16	99 322	99 503	33	23	60,71	65,92	0,33	0,23
17	99 289	99 480	77	39	59,73	64,94	0,77	0,39
18	99 212	99 441	90	35	58,77	63,96	0,91	0,35
19	99 122	99 407	123	34	57,83	62,99	1,24	0,34
20	98 999	99 373	155	60	56,90	62,01	1,57	0,60
21	98 844	99 313	142	15	55,99	61,04	1,44	0,15

5

40	96 600	98 433	147	85	38,05	42,49	1,53	0,86
41	96 453	98 348	144	110	37,11	41,53	1,49	1,12
42	96 309	98 239	208	100	36,16	40,58	2,16	1,02
43	96 101	98 138	181	110	35,24	39,62	1,89	1,12
44	95 919	98 029	205	112	34,31	38,66	2,14	1,15
45	95 715	97 916	190	153	33,38	37,70	1,98	1,57
46	95 525	97 763	256	172	32,44	36,76	2,68	1,76
47	95 268	97 591	256	160	31,53	35,83	2,68	1,64
48	95 013	97 431	324	191	30,61	34,88	3,41	1,96
49	94 689	97 240	310	197	29,72	33,95	3,28	2,03
50	94 379	97 042	324	233	28,81	33,02	3,43	2,40
51	94 055	96 810	387	265	27,91	32,10	4,11	2,74
52	93 668	96 545	332	255	27,02	31,18	3,54	2,64
53	93 336	96 290	461	293	26,12	30,27	4,94	3,04
54	92 875	95 997	504	343	25,25	29,36	5,42	3,58
55	92 371	95 653	546	342	24,38	28,46	5,91	3,57
56	91 825	95 311	583	362	23,52	27,56	6,35	3,80
57	91 242	94 949	647	400	22,67	26,66	7,09	4,22
58	90 595	94 549	593	435	21,83	25,77	6,55	4,60
59	90 002	94 115	713	554	20,97	24,89	7,92	5,89
60	89 289	93 560	797	543	20,13	24,04	8,93	5,81
61	88 492	93 017	853	543	19,31	23,17	9,64	5,83
62	87 639	92 475	911	626	18,49	22,31	10,39	6,77
63	86 728	91 848	1 200	781	17,68	21,45	13,84	8,50
64	85 528	91 068	1 359	795	16,92	20,63	15,89	8,73
65	84 168	90 273	1 356	763	16,19	19,81	16,11	8,45
66	82 812	89 509	1 349	883	15,44	18,98	16,29	9,86
67	81 463	88 627	1 572	897	14,69	18,16	19,30	10,12
68	79 891	87 730	1 746	1 070	13,97	17,34	21,86	12,19
69	78 145	86 660	1 869	1 056	13,27	16,55	23,91	12,19

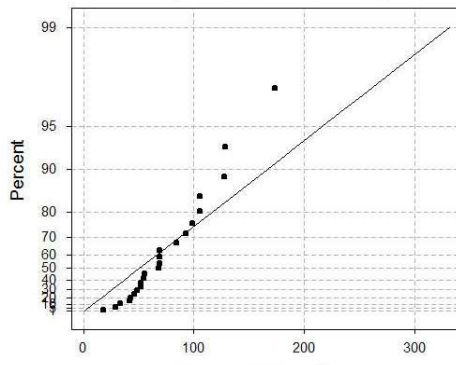
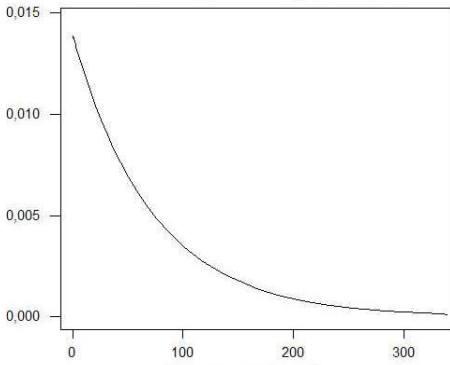
6

Ball Bearings Failure Data

ML Estimates - Complete Data

Exponential Probability

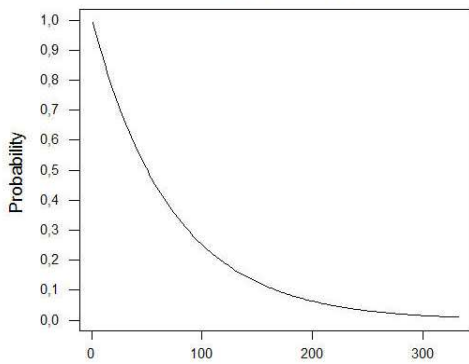
Probability Density Function



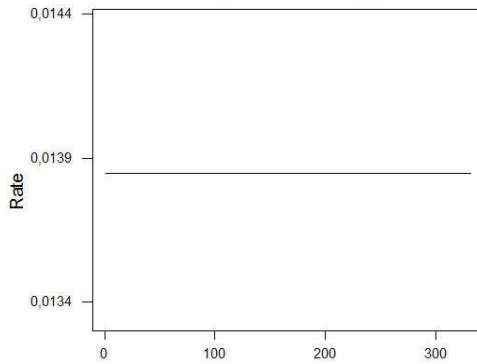
Shape 1
Scale 72,221
MTTF 72,221
Failure 23
Censor 0

Goodness of Fit
AD* 3,341

Survival Function



Hazard Function



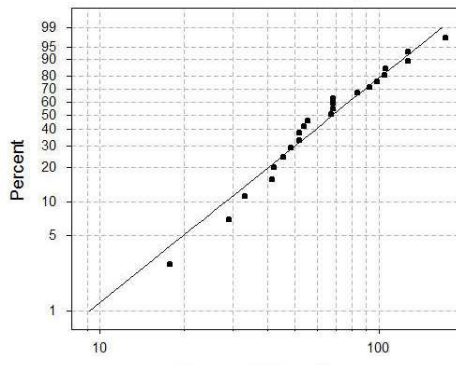
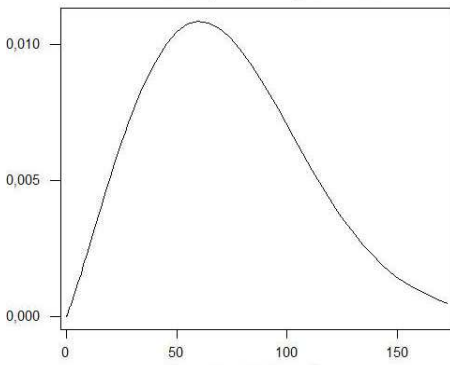
7

Ball Bearings Failure Data

ML Estimates - Complete Data

Weibull Probability

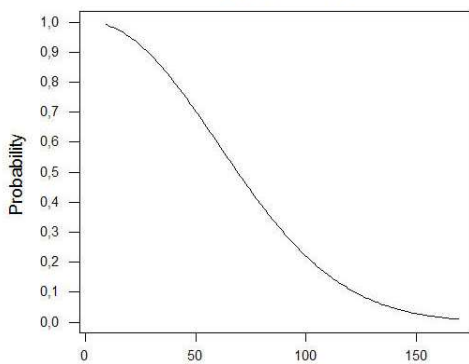
Probability Density Function



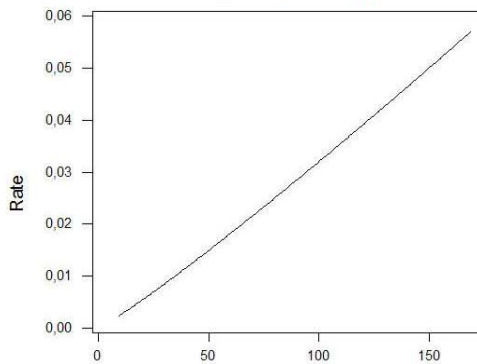
Shape 2,1018
Scale 81,875
MTTF 72,515
Failure 23
Censor 0

Goodness of Fit
AD* 0,802

Survival Function



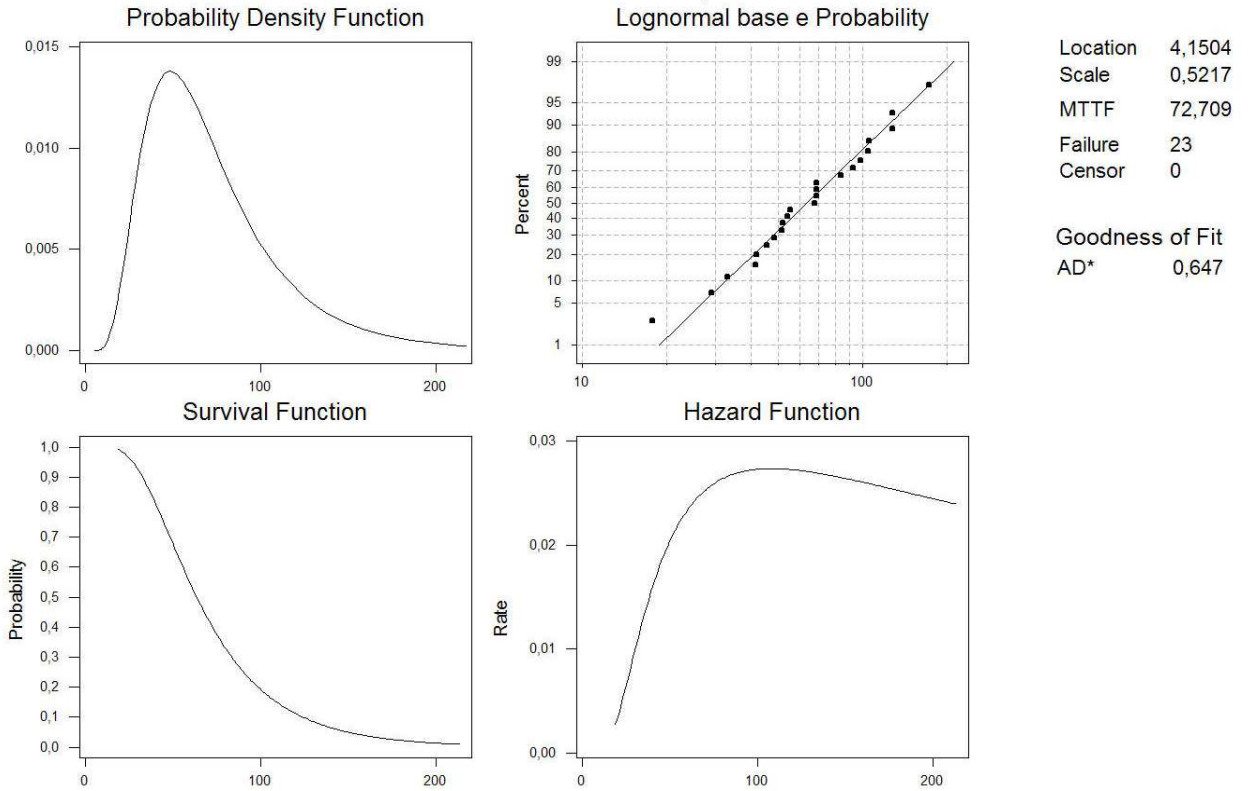
Hazard Function



8

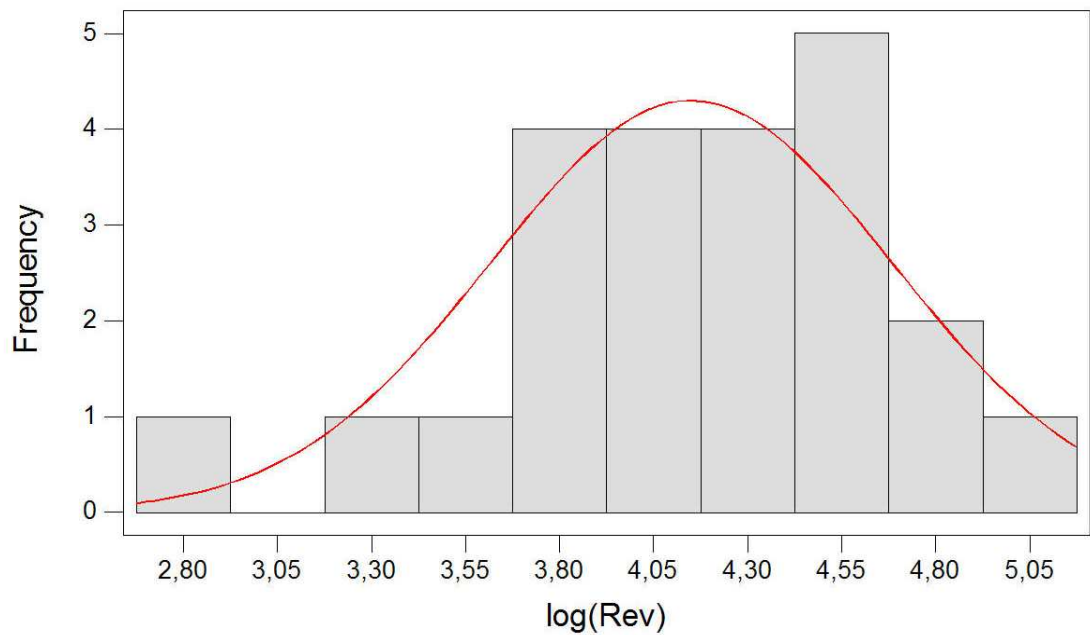
Ball Bearings Failure Data

ML Estimates - Complete Data



9

Histogram of log(Rev), with Normal Curve



10

AIRCONDITION FAILURES ON BOEING AIRPLANES (PROSCHAN, 1963)

MINITAB - Untitled

File Edit Manip Calc Stat Graph Editor Window Help

Worksheet 1 ***

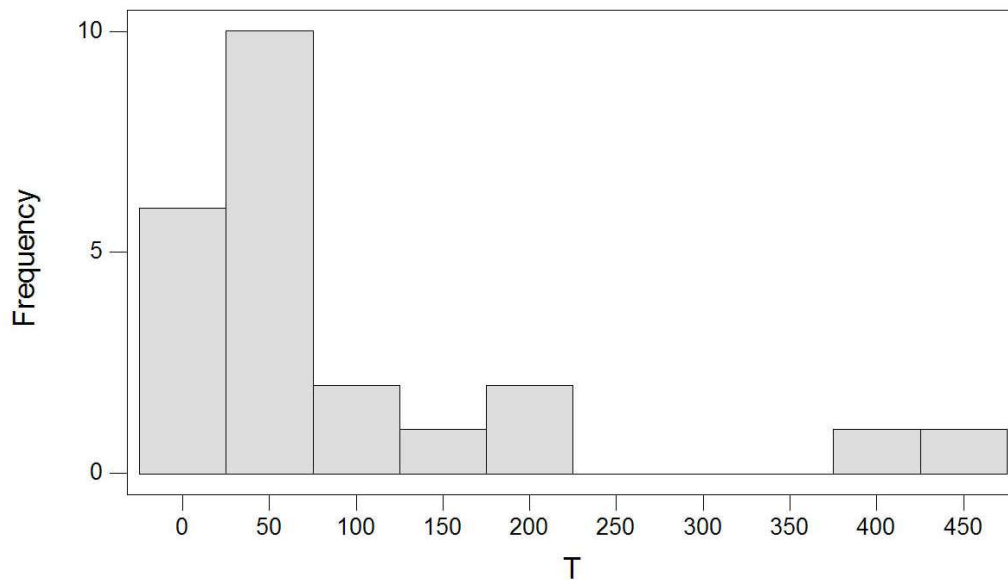
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17
	T	log(T)															
1	413	6.02345															
2	14	2.63906															
3	58	4.06044															
4	37	3.61092															
5	100	4.60517															
6	65	4.17439															
7	9	2.19722															
8	169	5.12990															
9	447	6.10256															
10	184	5.21494															
11	36	3.58352															
12	201	5.30330															
13	118	4.77068															
14	34	3.52636															
15	31	3.43399															
16	18	2.89037															
17	18	2.89037															
18	67	4.20469															
19	57	4.04305															
20	62	4.12713															
21	7	1.94591															
22	22	3.09104															
23	34	3.52636															
24																	
25																	
26																	
27																	
28																	
29																	
30																	
31																	
32																	

Current Worksheet: Worksheet 1

22:33

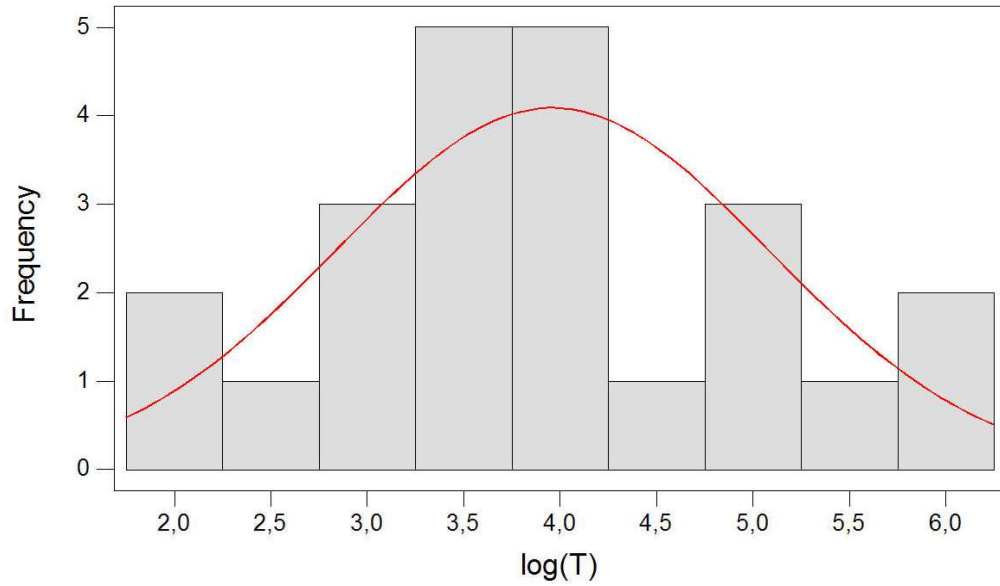
11

Histogram of T



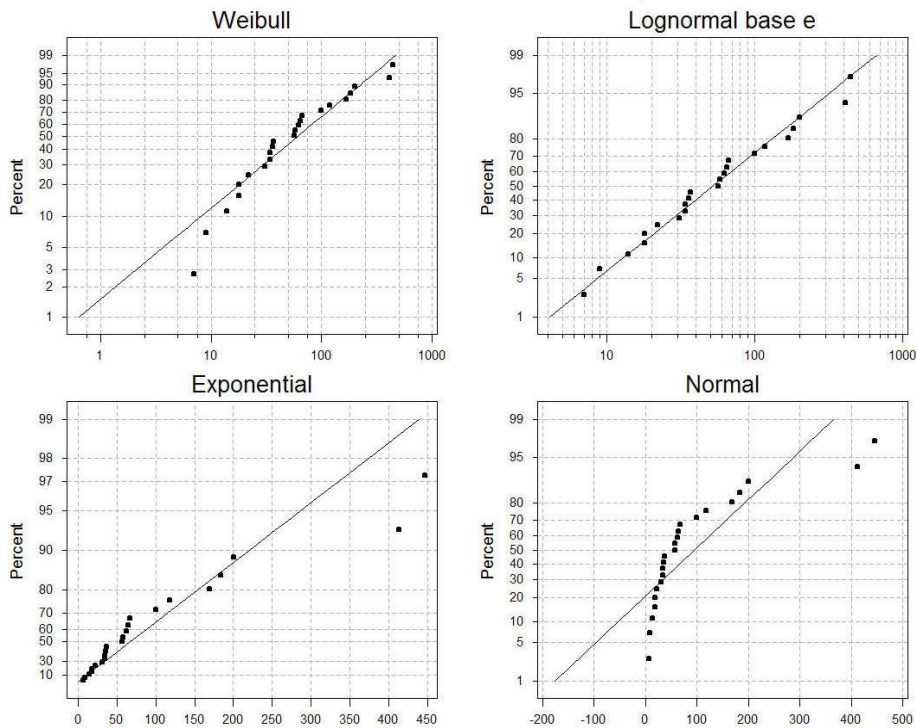
12

Histogram of log(T), with Normal Curve



Four-way Probability Plot for T

ML Estimates - Complete Data



Anderson-Darling (adj)

Weibull

0,999

Lognormal base e

0,675

Exponential

1,110

Normal

2,957