LECTURE WEEK 6

Spring 2005

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TMA4275 LIFETIME ANALYSIS

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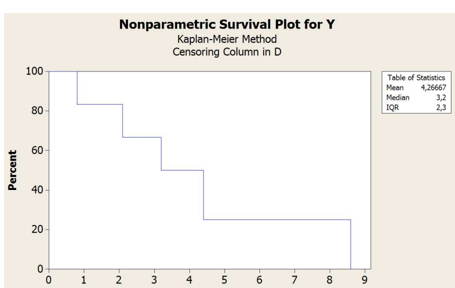
Nonparametric and parametric survival plots
for data from exponential example

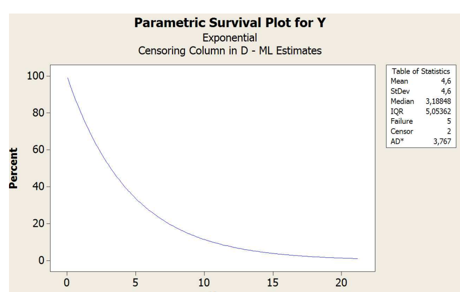
Worksheet 1 ***

↓	C1	C2
	Y	D
1	0,6	0
2	0,8	1
3	2,1	1
4	3,2	1
5	3,3	0
6	4,4	1
7	8,6	1

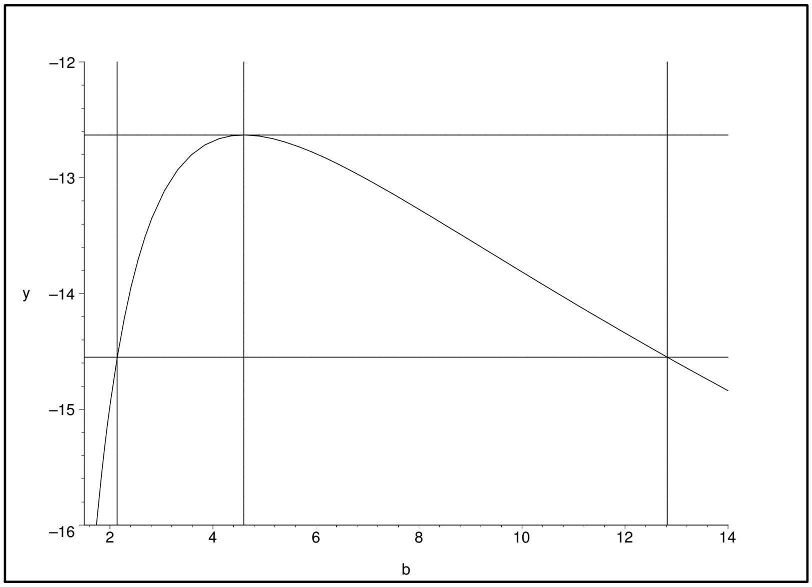
,



,



LOG-LIKELIHOOD FUNCTION



RIGHT CENSORED DATA FOR LECTURE WEEK 6 EXPONENTIAL AND WEIBULL MODELS

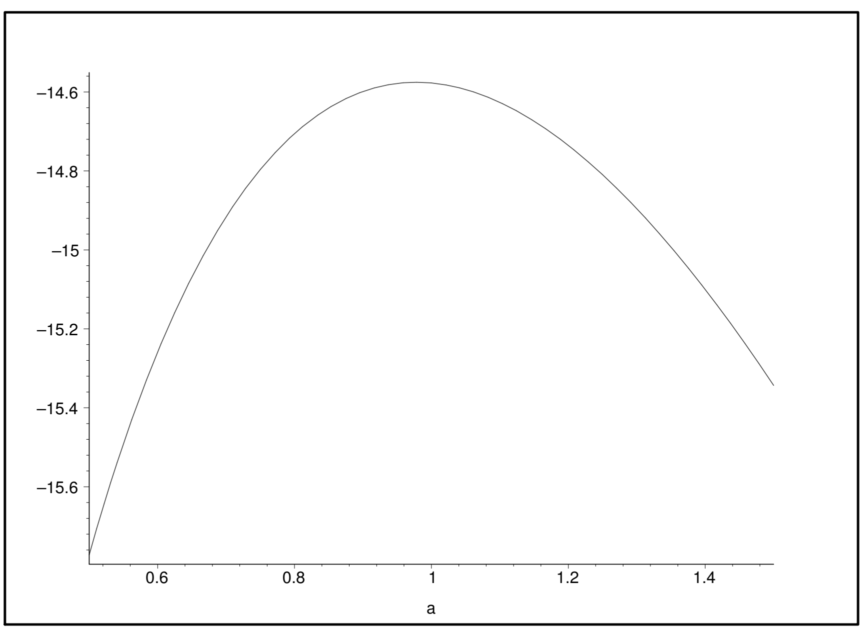
Row	C1	C2
1	0,35	1
2	0,50	0
3	0,75	0
4	1,00	1
5	1,30	1
6	1,80	1
7	3,00	0
8	3,15	0
9	4,85	0
10	5,50	1
11	5,50	0
12	6,25	0

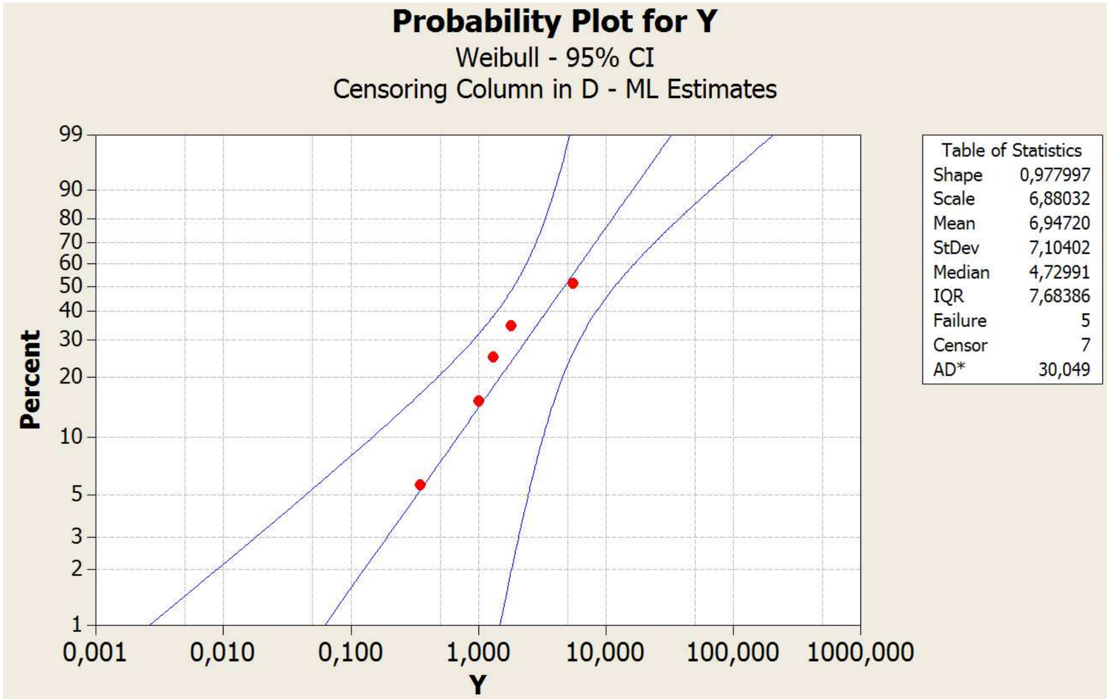
```
Variable:  C1
Censoring Information Count
Uncensored value 5
Right censored value 7
Censoring value:  C2 = 0

Estimation Method:  Maximum Likelihood Distribution:  Exponential

Parameter Estimates
                      Standard    95,0% Normal CI
Parameter   Estimate     Error    Lower     Upper
Shape        1,00000
Scale          6,790     3,037    2,826    16,313

Log-Likelihood = -14,577
```

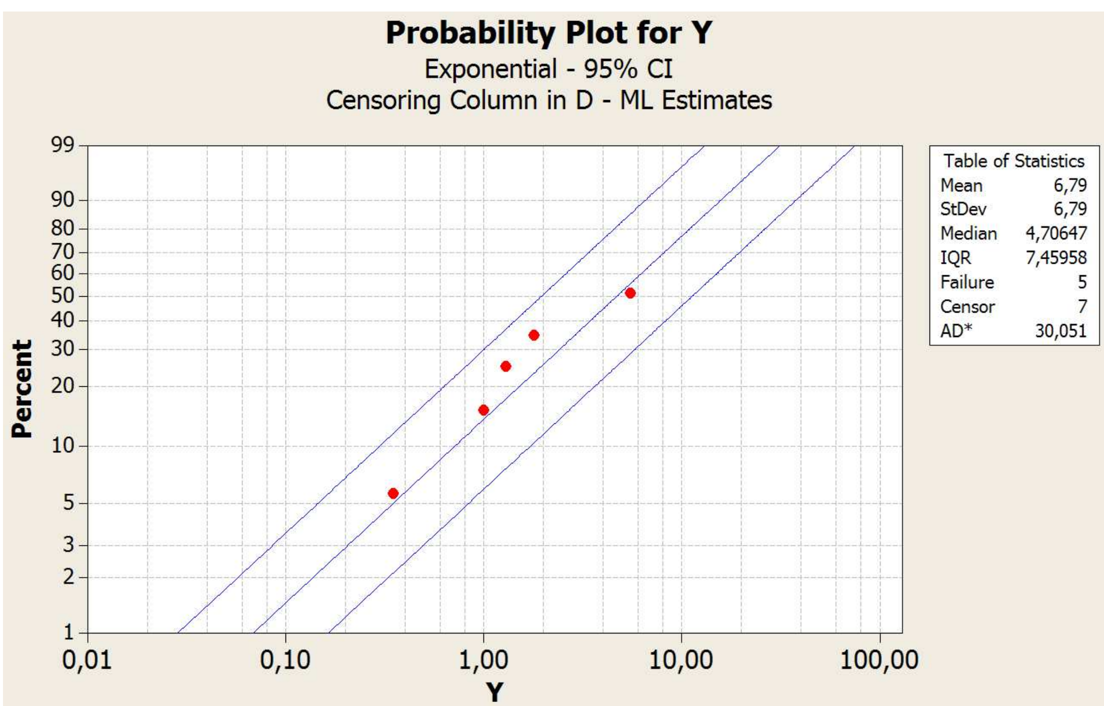
WEIBULL MODEL

```
Variable:  C1

Censoring Information
Count Uncensored value 5
Right censored value 7
Censoring value:  C2 = 0

Estimation Method:  Maximum Likelihood Distribution:  Weibull

Parameter Estimates
                        Standard        95,0% Normal CI
Parameter   Estimate       Error       Lower       Upper
Shape         0,9780      0,3694      0,4665      2,0504
Scale
              6,880       3,517       2,526      18,740

Log-Likelihood = -14,576
```


PROFILE LIKELIHOOD FOR SHAPE PARAMETER α IN WEIBULL DISTRIBUTION




Probability Plot for Y
Weibull - 95% CI
Censoring Column in D - ML Estimates
Percent
99
90
80
70
60
50
40
30
20
10
5
3
2
1
0,001
0,010
0,100
1,000
10,000
100,000
1000,000
Y
Table of Statistics
Shape 0,977997
Scale 6,88032
Mean 6,94720
StDev 7,10402
Median 4,72991
IQR 7,68386
Failure 5
Censor 7
AD* 30,049


Probability Plot for Y
Exponential - 95% CI
Censoring Column in D - ML Estimates
Percent
99
90
80
70
60
50
40
30
20
10
5
3
2
1
0,01
0,10
1,00
10,00
100,00
Y
Table of Statistics
Mean 6,79
StDev 6,79
Median 4,70647
IQR 7,45958
Failure 5
Censor 7
AD* 30,051

Method of obtaining probability plot points

main topic

Probability Plot creates an estimated cumulative distribution function (cdf) from your sample by plotting the value of each observation (including repeated values) against its estimated cumulative probability.

Estimated cumulative probability is calculated by one of the following formulas, according to what is selected in Tools > Options > Individual Graphs > Probability Plots (the default is median rank). For each formula, let n equal the number of observations and i equal the rank-order of each observation such that i = 1 for the smallest value and i = n for the largest.

Method	Formula
Median Rank (Benard)	$\frac{i - 0.3}{n + 0.4}$
Mean Rank (Herd-Johnson)	$\frac{i}{n + 1}$
Modified Kaplan-Meier (Hazen)	$\frac{i - 1/2}{n}$
Kaplan-Meier	$\frac{i}{n}$

The fitted line represents the cdf for the chosen theoretical distribution with the indicated parameters (either estimated or historical). The y-values (and in some cases the x-values) are transformed so that the fitted line is linear. Tick labels, however, remain consistent with the untransformed values. Thus, to the extent that the chosen distribution fits your data, the plotted points form a straight line.

The table below shows the transformations used for each distribution.

Distribution	X-coordinate	Y-coordinate (score)
Normal	data	$\Phi^{-1}(p)$
Lognormal	ln(data)	$\Phi^{-1}(p)$
3-parameter lognormal	ln(data - threshold)	$\Phi^{-1}(p)$
Gamma	ln(data)	$G^{-1}(p)$, k
3-parameter gamma	ln(data - threshold)	$G^{-1}(p)$, k
Exponential	ln(data)	ln(-ln(1 - p))
2-parameter exponential	ln(data - threshold)	ln(-ln(1 - p))
Smallest extreme value	data	ln(-ln(1 - p))
Weibull	ln(data)	ln(-ln(1 - p))
3-parameter Weibull	ln(data - threshold)	ln(-ln(1 - p))
Largest extreme value	data	-ln(-ln(p))
Logistic	data	ln(p / (1 - p))
Loglogistic	ln(data)	ln(p / (1 - p))
3-parameter loglogistic	ln(data - threshold)	ln(p / (1 - p))