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TMA4275 LIFETIME ANALYSIS

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Methods and Formulas – Accelerated Life Testing

Equation	Models	Residuals
Lifetime regression	Linear	Ordinary
Response variable	Arrhenius	Standardized
Error term	Inverse temp	Cox-Snell
	Loge (Power)	

Equation

Lifetime regression

The regression model estimates the percentiles of the failure time distribution:

$Y = \beta_0 + \beta_1 X + \sigma\varepsilon$

where:

Y = either failure time or log(failure time)

β_0 = y-intercept (constant)

β_1 = regression coefficient

X = predictor values (may be transformed)

σ = 1/shape (Weibull distribution) or scale (other distributions)

ε = random error term

Response variable

Depending on the distribution, Y = failure time or log (failure time):

- For the Weibull, exponential, lognormal, and loglogistic distributions, Y = log (failure time)
- For the normal, extreme value, and logistic distributions, Y = failure time

When Y = log (failure time), Minitab takes the antilog to display the percentiles on the original scale.

Error term

The value of the error distribution also depends on the distribution chosen.

- For the normal distribution, the error distribution is the standard normal distribution − normal (0,1). For the lognormal distribution, Minitab takes the log base e of the data and uses a normal distribution.
- For the logistic distribution, the error distribution is the standard logistic distribution − logistic (0, 1). For the loglogistic distribution, Minitab takes the log of the data and uses a logistic distribution.
- For the extreme value distribution, the error distribution is the standard extreme value distribution − extreme value (0, 1). For the Weibull distribution and the exponential distribution (a type of Weibull distribution), Minitab takes the log of the data and uses the extreme value distribution.

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Models

Linear $Y = \beta_0 + \beta_1 * \text{accelerating variable} + \sigma\varepsilon$

where:

- Y is the failure time or log failure time
- σ is the reciprocal of the shape parameter (Weibull distribution) or the scale parameter (other distributions)
- ε is the random error term

Arrhenius $Y = \beta_0 + \beta_1 * [11604.83/\text{Degrees Celsius} + 273.16)] + \sigma\varepsilon$

where:

- Y is the failure time or log failure time
- σ is the reciprocal of the shape parameter (Weibull distribution) or the scale parameter (other distributions)
- ε is the random error term

Inverse temp $Y = \beta_0 + \beta_1 * [1/(\text{Degrees Celsius} + 273.16)] + \sigma\varepsilon$

where:

- Y is the failure time or log failure time
- σ is the reciprocal of the shape parameter (Weibull distribution) or the scale parameter (other distributions)
- ε is the random error term

Loge (Power) $Y = \beta_0 + \beta_1 * \log(\text{accelerating variable}) + \sigma\varepsilon$

where:

- Y is the failure time or log failure time
- σ is the reciprocal of the shape parameter (Weibull distribution) or the scale parameter (other distributions)
- ε is the random error term

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Insulate.MTW ***

↓	C1	C2	C3	C4	C5-T	C6	C7	C8	C9	C10	C11
	Temp	ArrTemp	Plant	FailureT	Censor	Design	NewTemp	ArrNewT	NewPlant		
1	170	26,1865	1	343	F	80	80	32,8600	1		
2	170	26,1865	1	869	F	100	80	32,8600	2		
3	170	26,1865	1	244	C		100	31,0988	1		
4	170	26,1865	1	716	F		100	31,0988	2		
5	170	26,1865	1	531	F						
6	170	26,1865	1	738	F						
7	170	26,1865	1	461	F						
8	170	26,1865	1	221	F						
9	170	26,1865	1	665	F						
10	170	26,1865	1	384	C						
11	170	26,1865	2	394	C						
12	170	26,1865	2	369	F						
13	170	26,1865	2	366	F						
14	170	26,1865	2	507	F						
15	170	26,1865	2	461	F						
16	170	26,1865	2	431	F						
17	170	26,1865	2	479	F						
18	170	26,1865	2	106	F						
19	170	26,1865	2	545	F						
20	170	26,1865	2	536	F						
21	150	27,4242	1	2134	C						
22	150	27,4242	1	2746	F						
23	150	27,4242	1	2859	F						
24	150	27,4242	1	1826	C						

MINITAB Help

File Edit Bookmark Options Help

Help Topics | Back | Print | << | >> | Glossary | Exit

Example of Accelerated Life Testing

main topic   interpreting results   session command   see also

Suppose you want to investigate the deterioration of an insulation used for electric motors. The motors normally run between 80 and 100° C. To save time and money, you decide to use accelerated life testing.

First you gather failure times for the insulation at abnormally high temperatures – 110, 130, 150, and 170° C – to speed up the deterioration. With failure time information at these temperatures, you can then extrapolate to 80 and 100° C. It is known that an Arrhenius relationship exists between temperature and failure time.

To see how well the model fits, you will draw a probability plot based on the standardized residuals.

1 Open the worksheet INSULATE.MTW.
2 Choose **Stat > Reliability/Survival > Accelerated Life Testing**.
3 In **Variables/Start variables**, enter **FailureT**. In **Accelerating variable**, enter **Temp**.
4 From **Relationship**, choose **Arrhenius**.
5 Click **Censor**. In **Use censoring columns**, enter **Censor**, then click **OK**.
6 Click **Graphs**. In **Enter design value to include on plot**, enter **80**. Click **OK**.
7 Click **Estimate**. In **Enter new predictor values**, enter **Design**, then click **OK** in each dialog box.

Session window output

Regression with Life Data: FailureT versus Temp

```
Response Variable:  FailureT

Censoring Information               Count
Uncensored value                       66
Right censored value                   14
Censoring value:  Censor = C

Estimation Method:  Maximum Likelihood
Distribution:  Weibull
Transformation on accelerating variable:  Arrhenius
```

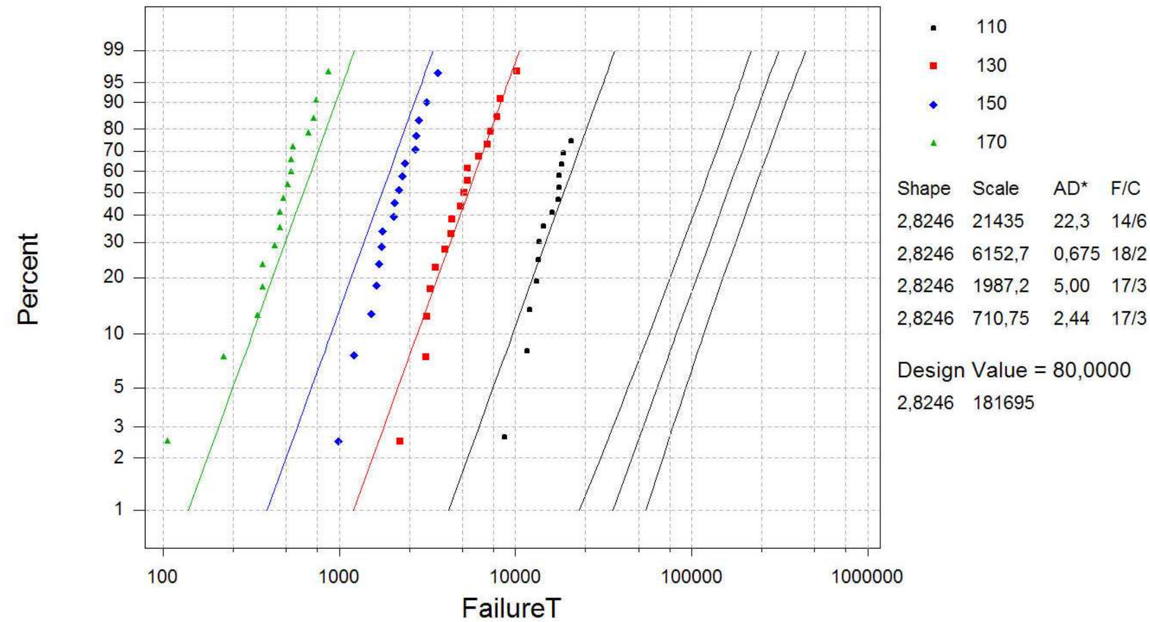


```
Regression Table
                          Standard                         95.0% Normal CI
Predictor       Coef         Error       Z      P        Lower        Upper
Intercept   -15.1874        0.9862  -15.40  0.000     -17.1203     -13.2546
Temp         0.83072       0.03504   23.71  0.000      0.76204      0.89940
Shape         2.8246        0.2570                      2.3633       3.3760

Log-Likelihood = -564.693

Anderson-Darling (adjusted) Goodness-of-Fit

At each accelerating level
Level      Fitted Model
110                  *
130                  *
150                  *
170                  *

Table of Percentiles
                                             Standard        95.0% Normal CI
 Percent         Temp   Percentile           Error        Lower        Upper
      50      80.0000     159584.5        27446.85     113918.2     223557.0
      50     100.0000     36948.57        4216.511     29543.36     46209.94
```

Probability Plot (Fitted Arrhenius) for FailureT

Weibull Distribution - ML Estimates - 95,0% CI

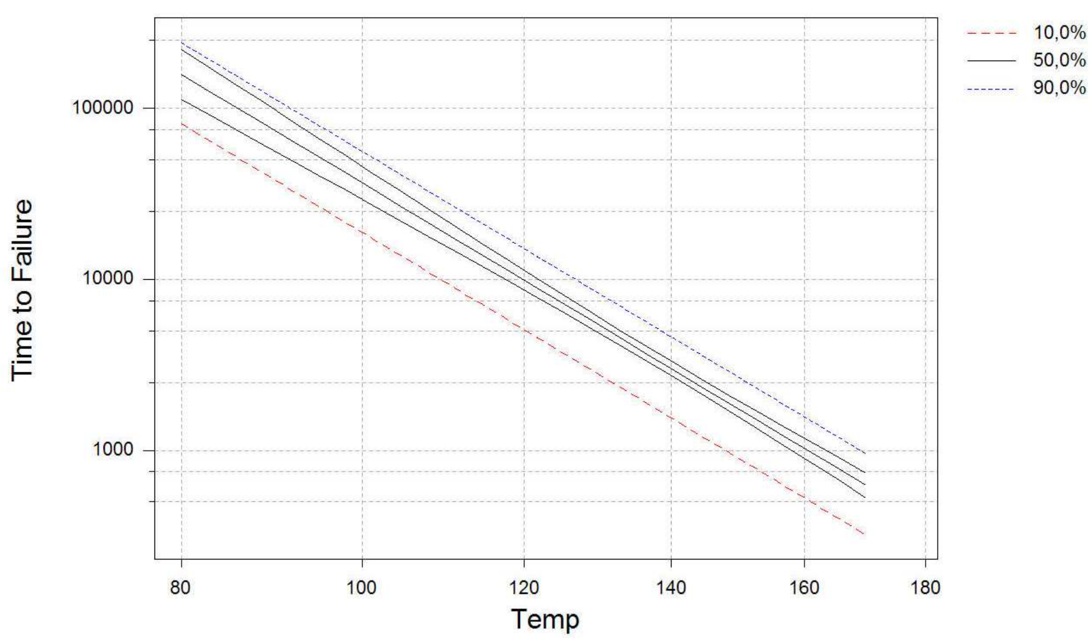
Censoring Column in Censor



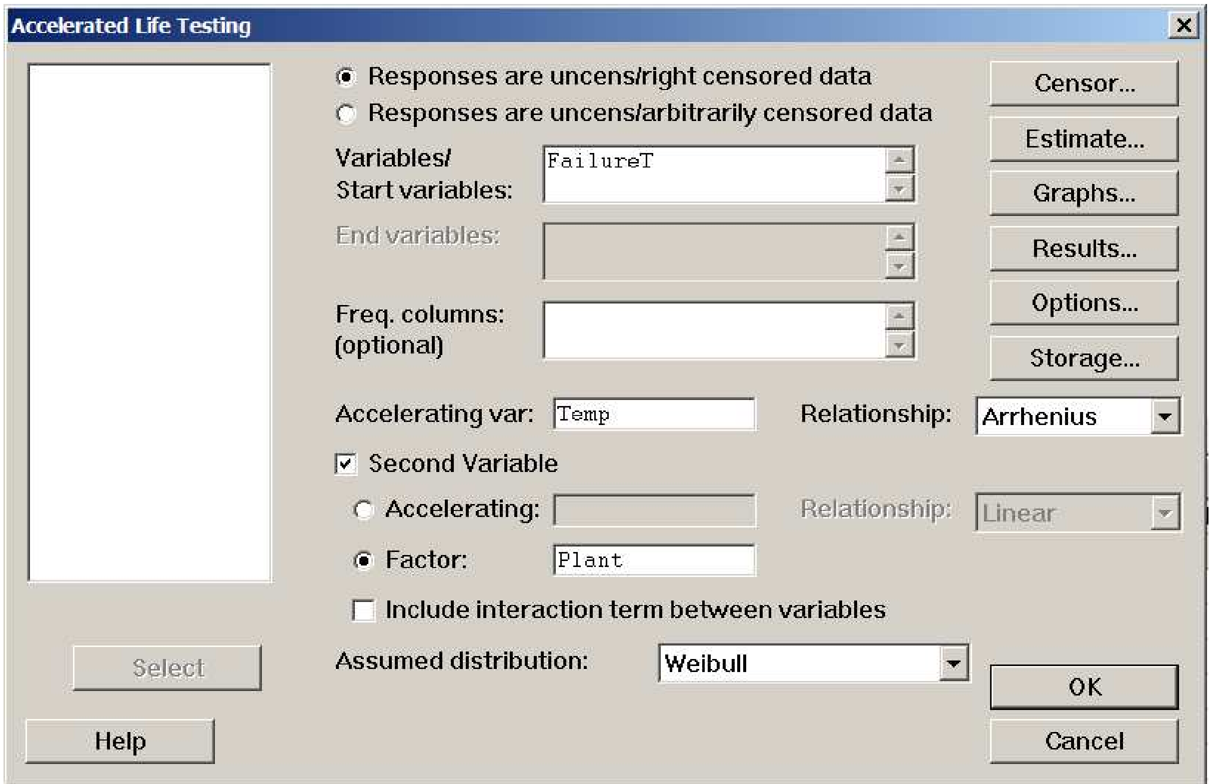
Relation Plot (Fitted Arrhenius) for FailureT

Weibull Distribution - ML Estimates - 95,0% CI

Censoring Column in Censor



ADDING THE FACTOR "PLANT":



RECURRENT EVENTS

RELIABILITY ANALYSIS:

- Repairable systems – events correspond to failures and repairs
- Manufactured products – events are warranty claims
- Example: Proschan(1963) – failures of airconditioners in Boeing airplanes

MEDICAL STUDIES:

- Repeated events, e.g. recurrence of infections, epileptic seizures, cancer
- Example: Aalen and Husebye (1991) – gastroenterology

PROSCHAN'S AIRCONDITION DATA

13 planes, 17 systems due to Major Overhaul

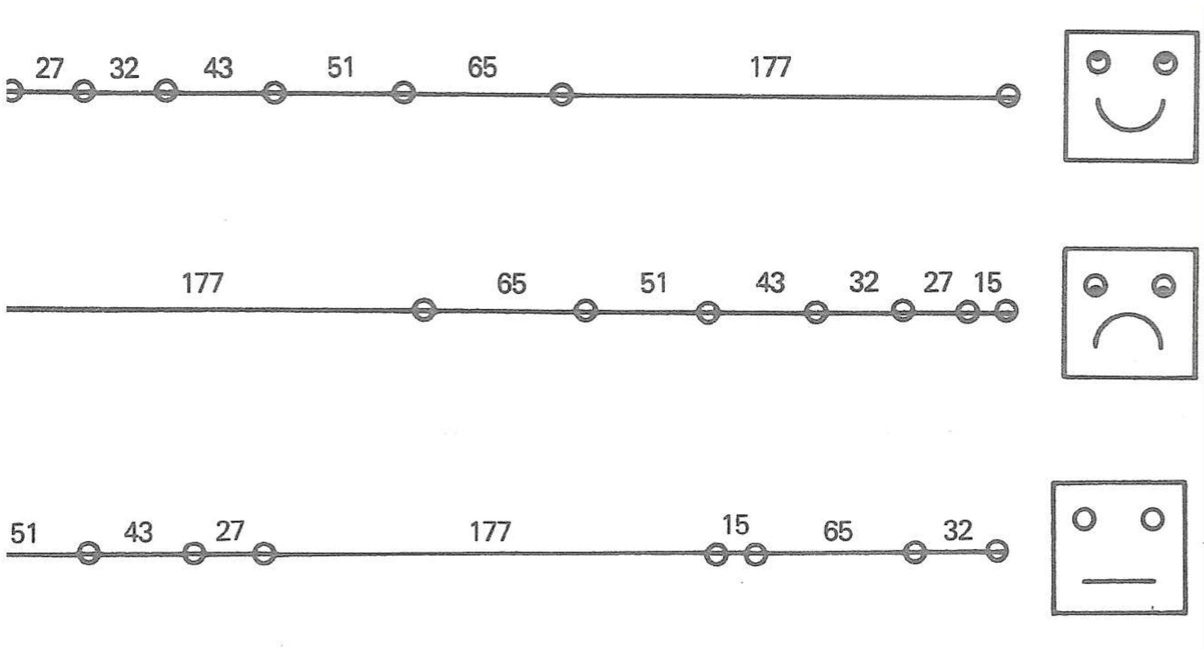
Plane	Interfailure times						
7907	194	15	41	29	33	181	
7908	413	14	58	37	100	65	9
	169	447	184	36	201	118	
7908 MO	34	31	18	18	67	57	62
	7	22	34				
7909	90	10	60	186	61	49	14
	24	56	20	79	84	44	59
	29	118	25	156	310	76	26
	44	23	62				
7909 MO	130	208	70	101	208		
⋮			⋮				

AALEN AND HUSEBYE'S DATA

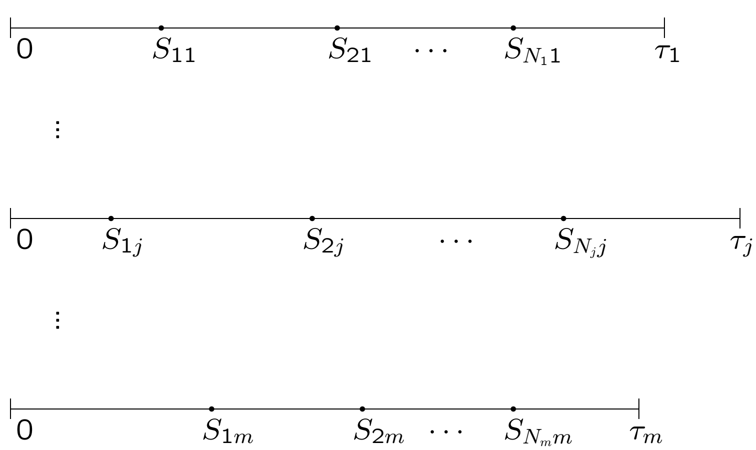
19 individuals, 1-9 events per individual

Individual	Observed periods					
1	112	145	39	52	21	34
	33	51	(54)			
2	206	147	(30)			
3	284	59	186	(4)		
4	94	98	84	(87)		
5	67	(131)				
6	124	34	87	75	43	38
	58	142	75	(23)		
7	116	71	83	68	125	(111)
8	111	59	47	95	(110)	
9	98	161	154	55	(44)	
10	166	56	(122)			
⋮						

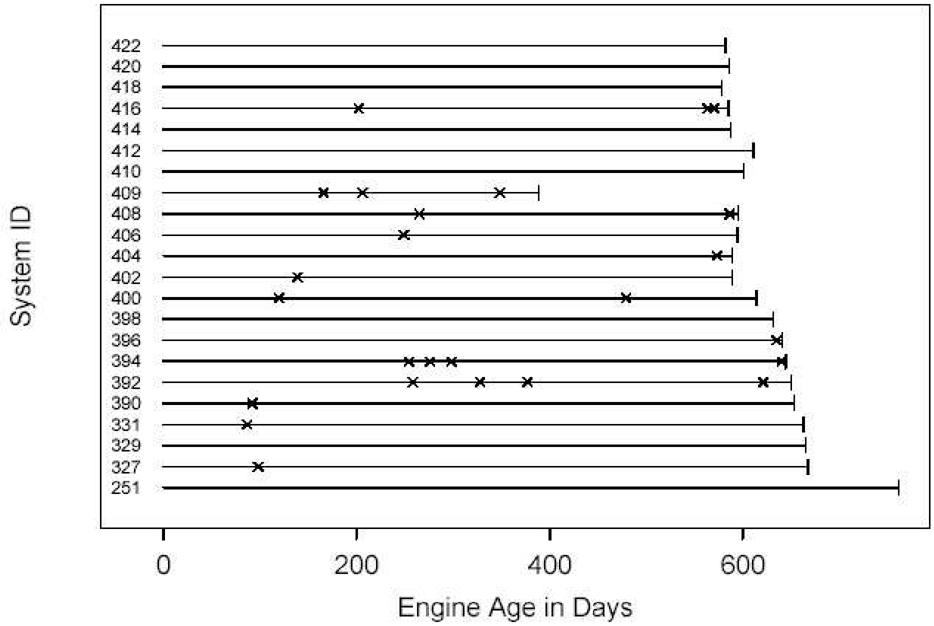
Three systems with the same interfailure times



TYPICAL DATA – RECURRENT EVENTS



Valve Seat Replacement Times Event Plot (Nelson and Doganaksoy 1989)

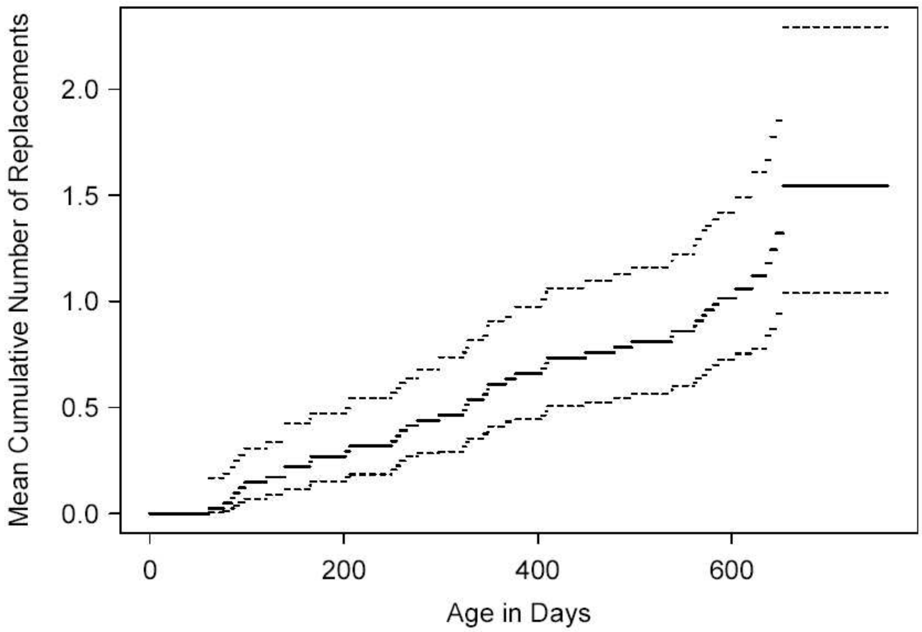


Valve Seat Replacement Times (Nelson and Doganaksoy 1989)

Data collected from valve seats from a fleet of 41 diesel engines (days of operation)

- Each engine has 16 valves
- Does the replacement rate increase with age?
- How many replacement valves will be needed in the future?
- Can valve life in these systems be modeled as a renewal process?

Estimate of Number of Valve Seat $\mu(t)$



Times of Unscheduled Maintenance Actions for a USS Grampus Diesel Engine

- Unscheduled maintenance actions caused by failure of imminent failure.
- Unscheduled maintenance actions are inconvenient and expensive.
- Data available for 16,000 operating hours.
- Data from Lee (1980).
- Is the system deteriorating (i.e., are failures occurring more rapidly as the system ages)?
- Can the occurrence of unscheduled maintenance actions be modeled by an HPP?

Cumulative Number of Unscheduled Maintenance Actions Versus Operating Hours for a USS Grampus Diesel Engine Lee (1980)