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TMA4275 LIFETIME ANALYSIS

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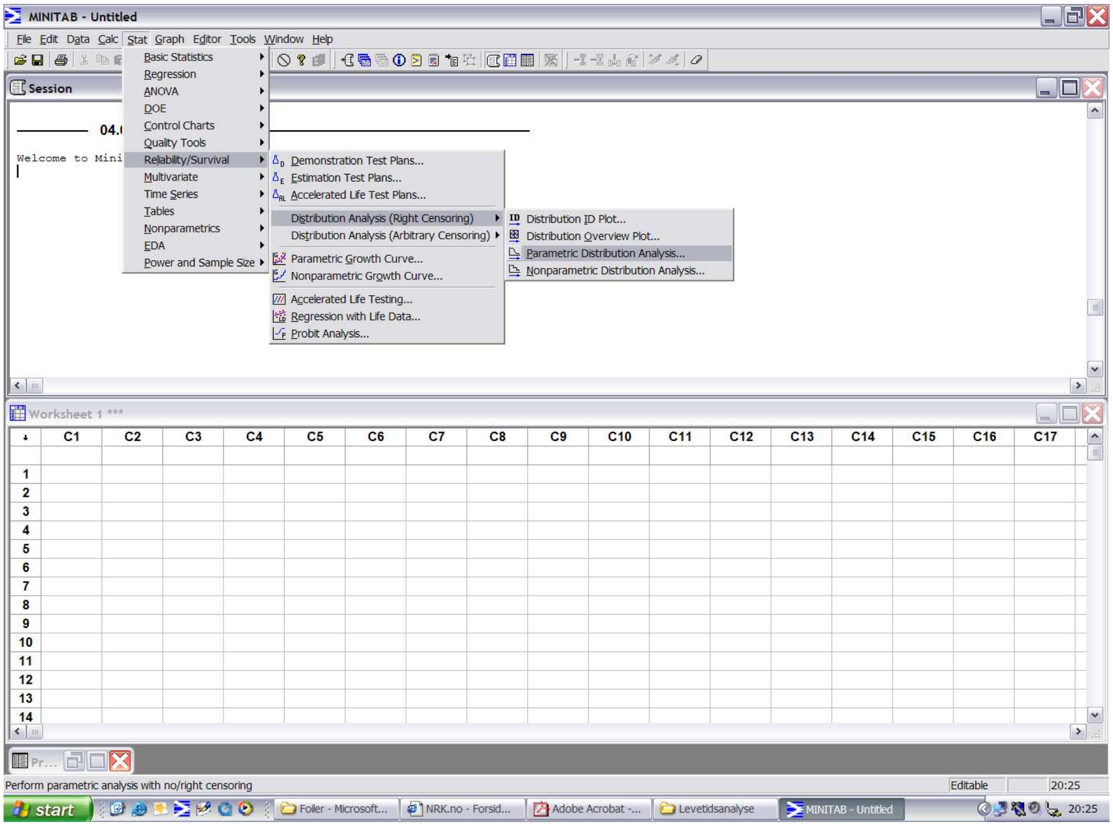
Why parametric models?

- **Complements nonparametric techniques.**
- **Parametric models can be described concisely with just a few parameters, instead of having to report an entire curve.**
- **It is possible to use a parametric model to extrapolate (in time) to the lower or upper tail of a distribution.**
- **Parametric models provide smooth estimates of failure-time distributions. In practice it is often useful to compare various parametric and nonparametric analyses of a data set.**

PARAMETRIC LIFETIME ANALYSIS IN MINITAB



DATA OPTIONS

RIGHT CENSORING:

Y_i	δ_i
Observed time	Cens. status 1: Lifetime 0: Censoring

ARBITRARY CENSORING:

Start variable A_i	End variable B_i	
1.7	1.7	Exact lifetime 1.7
2.0	*	Right censoring at time 2.0, i.e. lifetime is > 2.0
*	0.5	Left censoring at time 0.5, i.e. lifetime is < 0.5
1.0	1.5	Interval censoring: Lifetime between 1.0 and 1.5

LIKELIHOOD CONTRIBUTION

Obs. type	Start variable A_i	End variable B_i	Likelihood contribution
Exact lifetime	1.7	1.7	$f(1.7;\theta)$
Right censoring	2.0	*	$1 - F(2.0;\theta)$
Left censoring	*	0.5	$F(0.5;\theta)$
Interval censoring	1.0	1.5	$F(1.5;\theta) - F(1.0;\theta)$

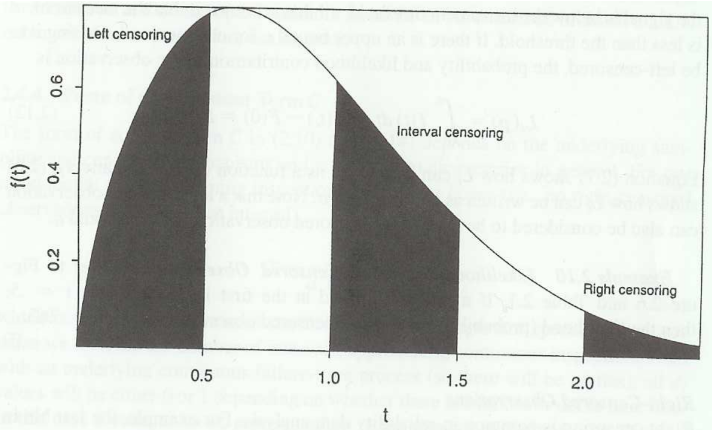
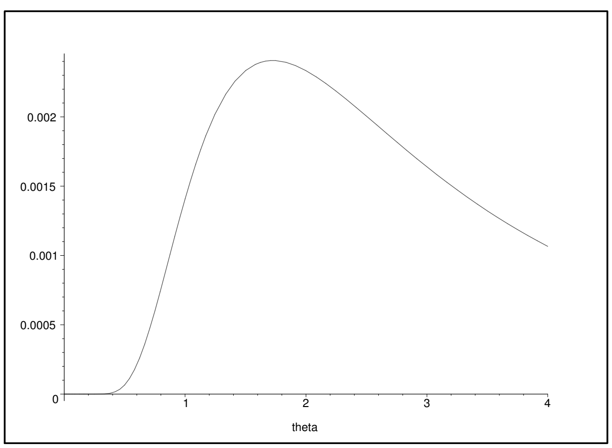


Figure 2.6. Likelihood contributions for different kinds of censoring.

LIKELIHOOD FOR MODEL $f(t;\theta) = (1/\theta)e^{-t/\theta}$

$$L(\theta) = (\frac{1}{\theta}e^{-1.7/\theta}) \cdot (e^{-2.0/\theta}) \cdot (1 - e^{-0.5/\theta}) \cdot (e^{-1.0/\theta} - e^{-1.5/\theta})$$



Maximum likelihood estimate: $\hat{\theta} = 1.725$

ARBITRARY CENSORED DATA: MINITAB OUTPUT

Distribution Analysis, Start = A and End = B

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Variable Start: A  End: B

Censoring Information    Count
Uncensored value             1
Right censored value         1
Interval censored value      1
Left censored value          1

Estimation Method: Maximum Likelihood

Distribution:   Exponential


Parameter Estimates

                     Standard   95,0% Normal CI
Parameter  Estimate     Error     Lower    Upper
Mean        1,72529  0,998421  0,554978  5,36353

Log-Likelihood = -6,029

Goodness-of-Fit
Anderson-Darling (adjusted) = 4,933


Characteristics of Distribution

                                    Standard   95,0% Normal CI
                         Estimate      Error     Lower     Upper
Mean(MTTF)                1,72529   0,998421  0,554978   5,36353
Standard Deviation        1,72529   0,998421  0,554978   5,36353
Median                    1,19588   0,692053  0,384682   3,71771
First Quartile(Q1)       0,496336   0,287228  0,159657   1,54299
Third Quartile(Q3)        2,39177    1,38411  0,769363   7,43543
Interquartile Range(IQR)  1,89543    1,09688  0,609706   5,89244
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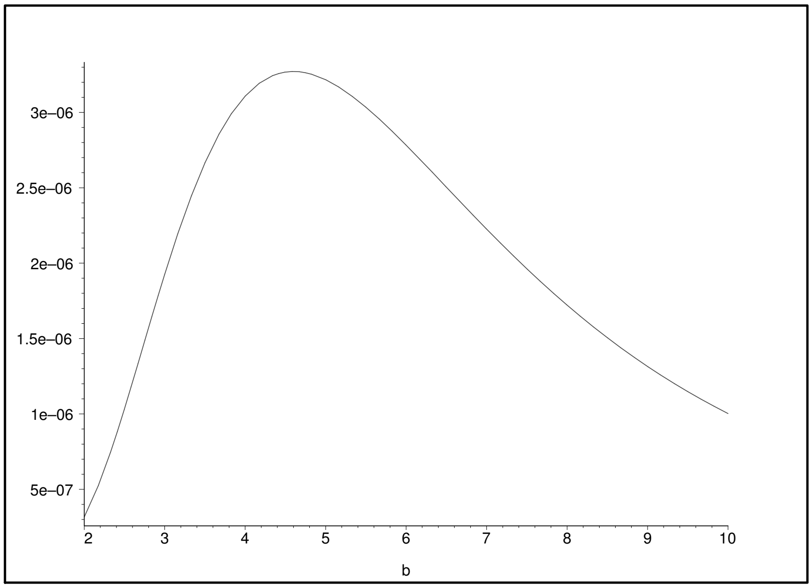
EXAMPLE: RIGHT CENSORED DATA

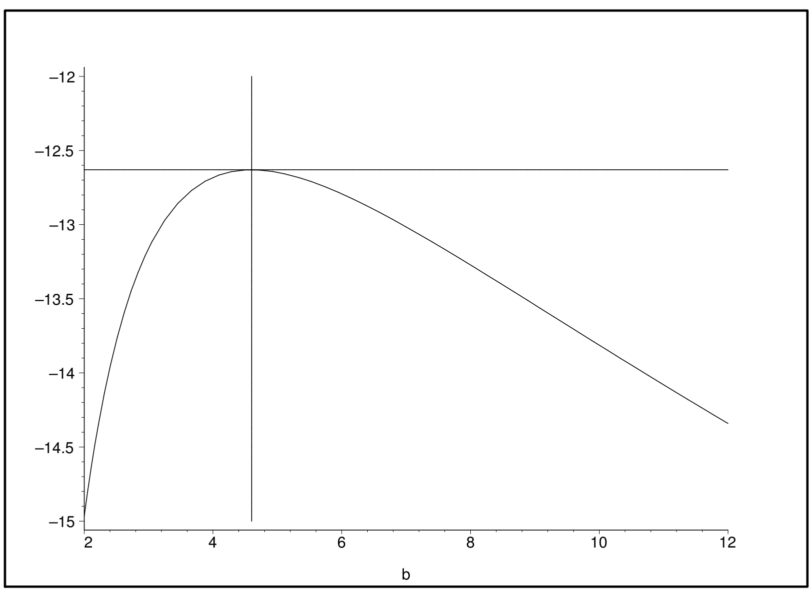
Worksheet 1 ***

↓	C1	C2
	Y	D
1	0,6	0
2	0,8	1
3	2,1	1
4	3,2	1
5	3,3	0
6	4,4	1
7	8,6	1

LIKELIHOOD FUNCTION FOR MODEL $f(t;\theta) = (1/\theta)e^{-t/\theta}$



LOG-LIKELIHOOD FUNCTION



Maximum likelihood estimate: $\hat{\theta} = 4.6$

Distribution Analysis: Y

Variable: Y

Censoring Information	Count
Uncensored value	5
Right censored value	2

Censoring value: D = 0

Estimation Method: Maximum Likelihood

Distribution: Exponential

Parameter Estimates

Parameter	Estimate	Standard Error	95,0% Normal CI Lower	Upper
Mean	4,6	2,05718	1,91465	11,0516

Log-Likelihood = -12,630

Goodness-of-Fit
Anderson-Darling (adjusted) = 3,767

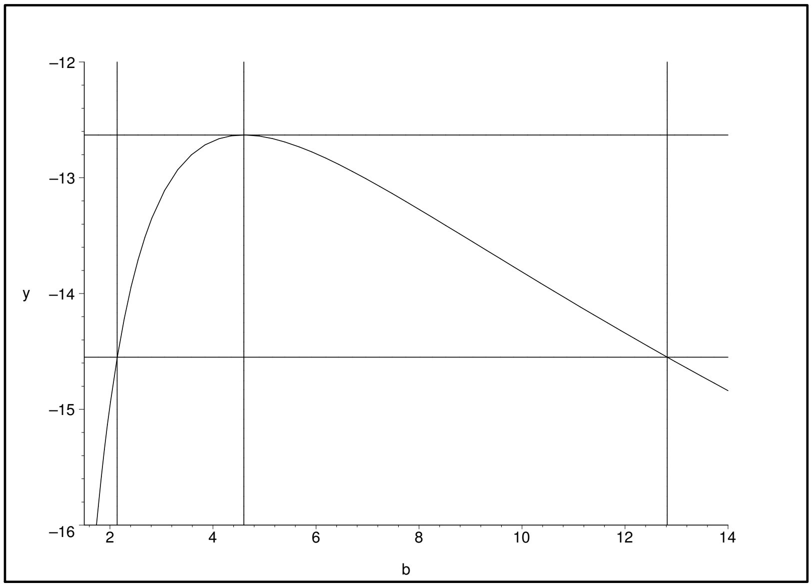
Characteristics of Distribution

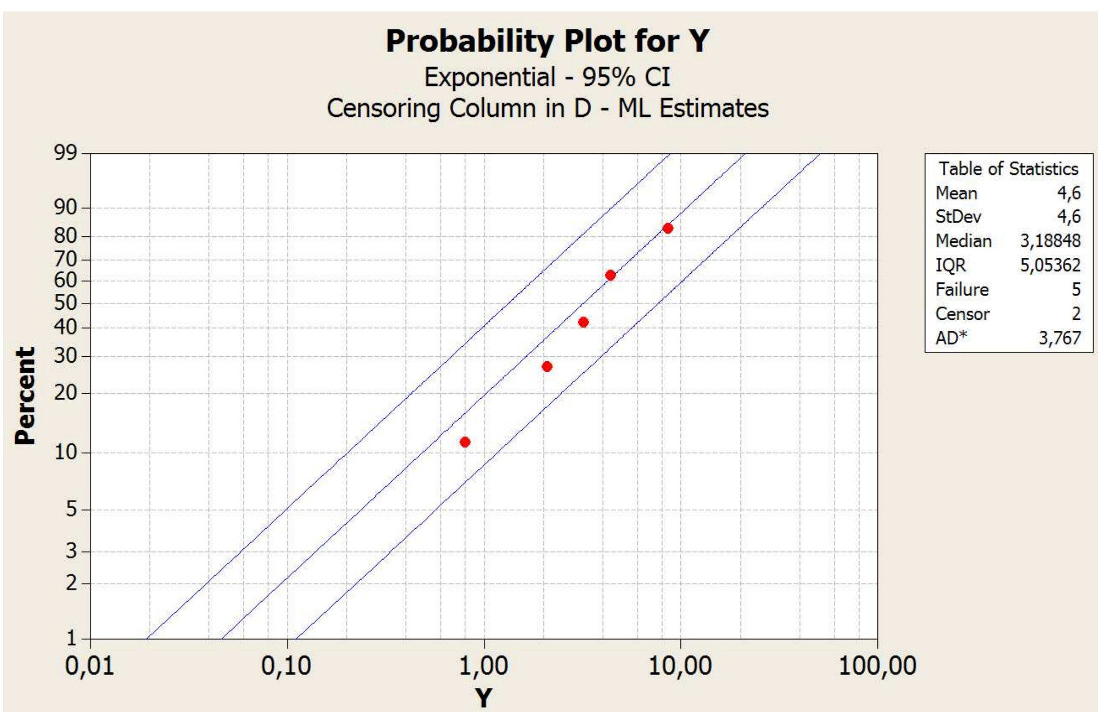
	Estimate	Standard Error	95,0% Normal CI Lower	Upper
Mean(MTTF)	4,6	2,05718	1,91465	11,0516
Standard Deviation	4,6	2,05718	1,91465	11,0516
Median	3,18848	1,42593	1,32713	7,66041
First Quartile(Q1)	1,32334	0,591815	0,550810	3,17936
Third Quartile(Q3)	6,37695	2,85186	2,65427	15,3208
Interquartile Range(IQR)	5,05362	2,26005	2,10346	12,1415

Table of Percentiles

Percent	Percentile	Standard Error	95,0% Normal CI Lower	Upper
1	0,0462315	0,0206754	0,0192429	0,111073
2	0,0929325	0,0415607	0,0386811	0,223273
3	0,140112	0,0626601	0,0583187	0,336624
4	0,187781	0,0839783	0,0781597	0,451150
5	0,235949	0,105520	0,0982086	0,566875
6	0,284627	0,127289	0,118470	0,683825
7	0,333825	0,149291	0,138947	0,802025
8	0,383555	0,171531	0,159646	0,921504
9	0,433829	0,194014	0,180572	1,04229
10	0,484658	0,216746	0,201728	1,16441
20	1,02646	0,459047	0,427241	2,46610
30	1,64070	0,733745	0,682907	3,94184
40	2,34980	1,05086	0,978051	5,64546
50	3,18848	1,42593	1,32713	7,66041
60	4,21494	1,88498	1,75437	10,1265
70	5,53827	2,47679	2,30518	13,3059
80	7,40341	3,31091	3,08151	17,7869
90	10,5919	4,73684	4,40864	25,4473
91	11,0765	4,95358	4,61037	26,6117
92	11,6184	5,19588	4,83588	27,9134
93	12,2326	5,47058	5,09155	29,3892
94	12,9417	5,78770	5,38669	31,0928
95	13,7804	6,16277	5,73577	33,1078
96	14,8068	6,62182	6,16301	35,5739
97	16,1302	7,21363	6,71382	38,7532
98	17,9953	8,04775	7,49015	43,2343
99	21,1838	9,47368	8,81728	50,8947

LOG-LIKELIHOOD FUNCTION





Nonparametric and parametric survival plots
for data from exponential example

Worksheet 1 ***

↓	C1	C2
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1	0,6	0
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5	3,3	0
6	4,4	1
7	8,6	1

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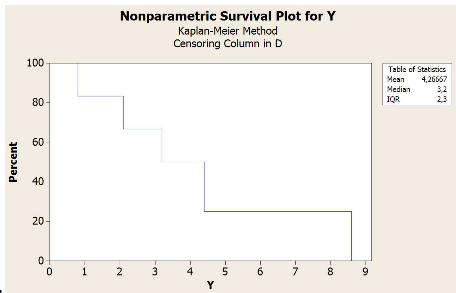

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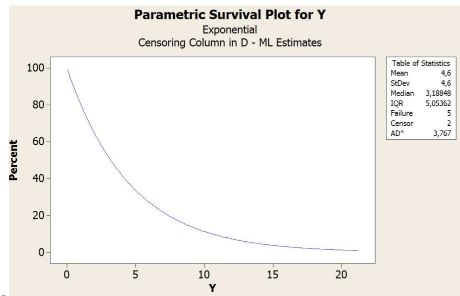

Motivation for the exponential distribution

- Simplest distribution used in the analysis of reliability data.
- Has the important characteristic that its hazard function is constant (does not depend on time t).
- Popular distribution for some kinds of electronic components (e.g., capacitors or robust, high-quality integrated circuits).
- This distribution would not be appropriate for a population of electronic components having failure-causing quality-defects.
- Might be useful to describe failure times for components that exhibit physical wearout only after expected technological life of the system in which the component would be installed.

Motivation for the Weibull distribution

- The theory of extreme values shows that the Weibull distribution can be used to model the minimum of a large number of independent positive random variables from a certain class of distributions.
  - Failure of the weakest link in a chain with many links with failure mechanisms (e.g. fatigue) in each link acting approximately independent.
  - Failure of a system with a large number of components in series and with approximately independent failure mechanisms in each component.
- The more common justification for its use is empirical: the Weibull distribution can be used to model failure-time data with a decreasing or an increasing hazard function.

Motivation for lognormal distribution

- The lognormal distribution is a common model for failure times.
- It can be justified for a random variable that arises from the product of a number of identically distributed independent positive random quantities (remember central limit theorem for sum of normals).
- It has been suggested as an appropriate model for failure time caused by a degradation process with combinations of random rates that combine multiplicatively.
- Widely used to describe time to fracture from fatigue crack growth in metals.
- Useful in modeling failure time of a population electronic components with a decreasing hazard function (due to a small proportion of defects in the population).
- Useful for describing the failure-time distribution of certain degradation processes.