Spring 2006

TMA4275 LIFETIME ANALYSIS

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GOALS

After finishing this course you should ....

- **know the most common concepts and distributions from lifetime modeling**
- **be able to use graphical methods for description and comparison of lifetime data**
- **be able to use statistical methods for statistical inference (estimation, confidence interval, hypothesis testing) of lifetime data**
- **be able to analyze lifetime data by using computer software (MINITAB)**

About the course

The course gives an introduction to stochastic modelling and statistical methods for use in lifetime data analysis, with particular view to applications in reliability analysis and medicine.

The lectures are based on knowledge from TMA4240/TMA4245 Statistics or equivalent. It will be an advantage to have taken one of the courses TPK4120 Industrial safety and reliability, TMA4260 Industrial statistics, and TMA4255 Design of experiments and applied statistical methods.

Contents: Basic concepts in lifetime modelling. Censored observations. Nonparametric estimation and graphical plotting for lifetime data (Kaplan-Meier, Nelson-plot). Estimation and testing in parametric lifetime distributions. Analysis of lifetimes with covariates. (Cox-regression, accelerated lifetime testing). Modelling and analysis of recurrent events. Nonhomogeneous Poisson-processes. Nelson-Aalen estimators. Bayesian lifetime analysis.

- Weekly hours: Spring: 4F+1Ø+7S = 7,5 SP
- Course type: Lectures and exercises with the use of a computer (MINITAB). Lectures may be given in English. Portfolio assessment is the basis for the grade awarded in the course. This portfolio comprises a written final examination 80% and selected parts of the exercises 20%. The results for the constituent parts are to be given in %-points, while the grade for the whole portfolio (course grade) is given by the letter grading system. Retake of examination may be given as an oral examination.

Lecturer

Professor Bo Lindqvist, room 1129, Sentralbygg II. Tlf. (735)93532
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Exercise lab teacher

Research assistant Anastasia Ushakova, room 1126, Sentralbygg II. Tlf. (735)91671
Office hours: To be announced.
Email: anastasi@math.ntnu.no

Course book

The main source will be the book Rausand & Høyland: System Reliability Theory: Models, Statistical Methods, and Applications, 2nd Edition. Wiley 2004.

Alternatively you may use the 1st edition: Høyland & Rausand: System Reliability Theory: Models and Statistical Methods, Wiley 1994.

Notes/copies about certain topics will be handed out. Foils from the lectures can be downloaded as pdf-files from this website.

Curriculum

THE FINAL CURRICULUM FOR spring 2005 can be found here.

Exercises

Some exercises (including the obligatory ones) require use of the statistics computer package MINITAB, see
http://www.ntnu.no/itea.info/programvare/minitab.html
Note that NTNU has an unlimited site licence for Windows and Macintosh for installation of MINITAB on NTNUs area and on private machines of students and staff. MINITAB is also available on computer labs Gombe (Realfagbygget R90, 24 machines) and Chobe (Realfagbygget R91, 12 machines).

Final exam:

May 26, 2006. Written. 4 hours.
Permitted aids: A. All printed or handwritten aids permitted. All calculators permitted.

PRELIMINARY CURRICULUM AND LECTURE PLAN

Week No.	Literature H & R	Literature R & H (2nd ed.)	Topic	Comment H & R	Comment R & H
3	2.1-2.4-2.7, 2.9-2.11, 2.14, 2.17 Notes on "Log-location-scale…" (Lecture week 2).	2.1-2.14, 2.17, 2.20 Notes on "Log-location-scale…" (Lecture week 2).	Probability distributions for lifetimes. Fundamental properties. Important distributions and properties.	Only main results in 2.14 are covered.	Only main results in 2.17 are covered.
4-5	Ch. 9 The note "About the exponential distribution …" (Lecture week 3).	11.1-11.3, 11.5 The note "About the exponential distribution …" (Lecture week 3).	Lifetime data. Censoring. Nonparametric methods. Plotting (TTT, Kaplan-Meier, Nelson-Aalen.)		
6-8	Notes on "Likelihood construction" and "On parametric inference…." (Lecture week 5).	11.4 Notes on "Likelihood construction" and "On parametric inference…." (Lecture week 5).	Parametric estimation and testing. Maximum likelihood. Information matrix. Confidence intervals. Probability plots (MINITAB).		
9-10			No lectures		
11-13	Ch. 10. Notes on "Survival regression" (Lecture week 8) and "Medical study" (Lecture week 9).	Ch. 12. Notes on "Survival regression" (Lecture week 8) and "Medical study" (Lecture week 9).	Regression methods. Covariates. Weibull regression. Cox-regression. Accellerated lifetime testing.	P. 428 and rest of chapter 10 are not covered.	Example 12.2 page 532 and rest of Ch. 12 are not covered.

14 and 16-17	Ch. 7	Ch. 7	Point processes. Recurrent events. Repairable systems. Poisson processes and renewal. Modelling and statistical analysis of data. Likelihood-methods.	Only selected parts of 7.3 are treated in class. From pages 293-313 are just "Superimposed …" s. 301-302 covered.	The following is covered (not always in detail): 7.1 only to 7.1.3; most of 7.2, but only to 7.2.6; 7.3 to 7.3.4; but in addition 7.3.8; most of 7.4. Pooled versions of Laplace and Mil-Hdbk tests (as used by MINITAB).
15			Easter vacation		
18	Ch. 11	Ch. 13	Bayesian lifetime analysis	Selected parts of 11.1-11.5 are covered.	Selected parts of 13.1-13.5 are covered.
19	Earlier exams		Review		

RELIABILITY

Common technical definition of reliability:

The probability that a system or a component will perform its intended task, under given operational conditions, for a specified time period.

LIFE TIMES (SURVIVAL DATA)

- **Time to failure of a component or a system**
- **Number of cycles to failure (fatigue testing)**
- **Time to next epileptic seizure for patient**
- **Times of failure and repair of a machine**

WHY COLLECT AND ANALYZE LIFETIME/SURVIVAL/RELIABILITY DATA?

- **Assess reliability of a system/component/product**
- **Compare two or more products with respect to reliability**
- **Predict product reliability in the design phase**
- **Predict warranty claims for a product in the market**

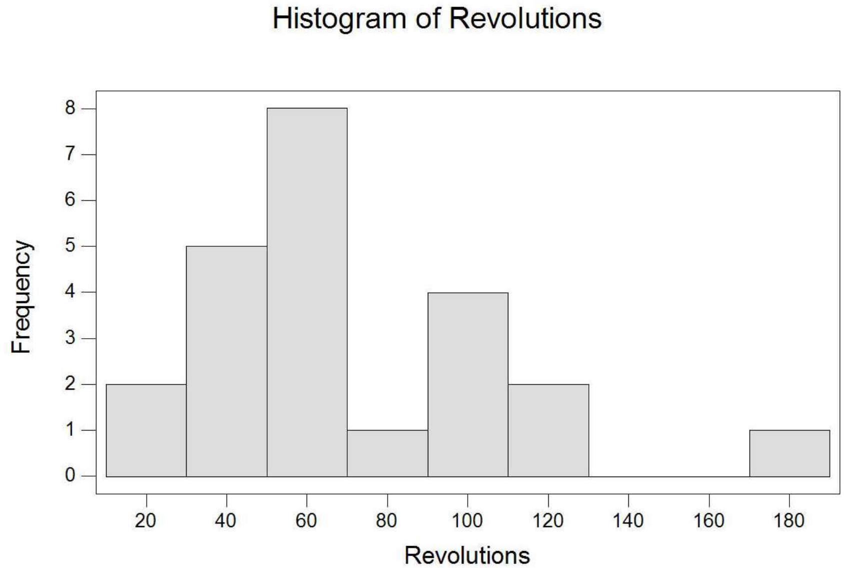
SPECIAL ASPECTS OF LIFETIME ANALYSIS

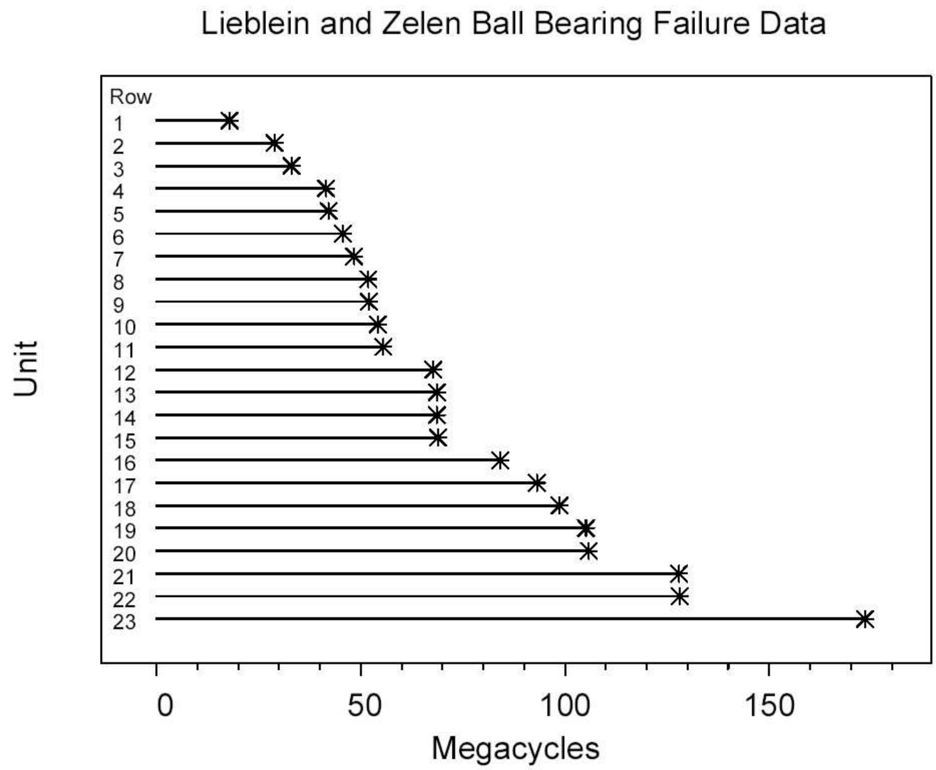
- Definition of starting time and failure time are difficult
- Definition of time scale (operation time, calendar time, number of cycles)
- Censored data (what do we do with units that have not failed within the observation period?)
- Effect of environmental conditions
- What if a unit fails of another cause than the one we would like to study? ("competing risks")
- Recurrent events – what if the system can fail several times; how to analyze recurring stages of a disease?

BALL BEARINGS FAILURE DATA

Data: Millions of revolutions to fatigue failure for 23 units

17,88	28,92	33,00	41,52	42,12	45,60	48,40	51,84
51,96	54,12	55,56	67,80	68,64	68,64	68,88	84,12
93,12	98,64	105,12	105,84	127,92	128,04	173,40	





re Tools

IC Data (Meeker, 1987)

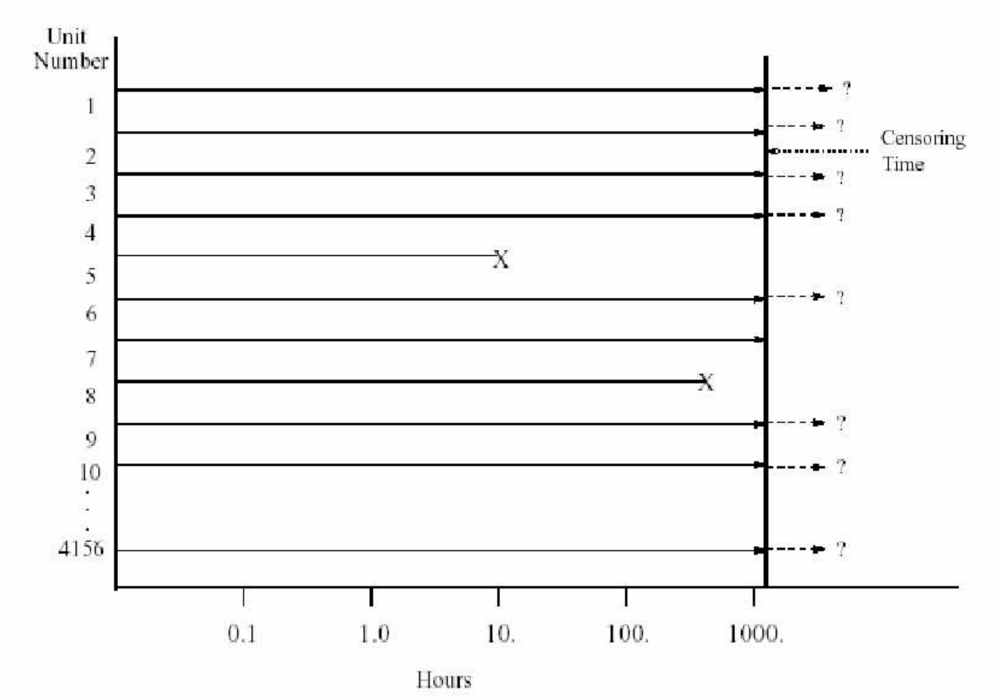
- Integrated circuit failure times in hours
  - *n* = 4156 ICs tested for 1,370 hours at 80° C and 80% relative humidity
  - There were 28 failures
  - When the test ended at 1,370 hours, 4128 units were still running

.10	.10	.15	.60	.80	.80
1.20	2.5	3.0	4.0	4.0	6.0
10.0	10.0	12.5	20.	20.	43.
43.	48.	48.	54.	74.	84.
94.	168.	263.	593.		

TYPICAL PROBLEMS:

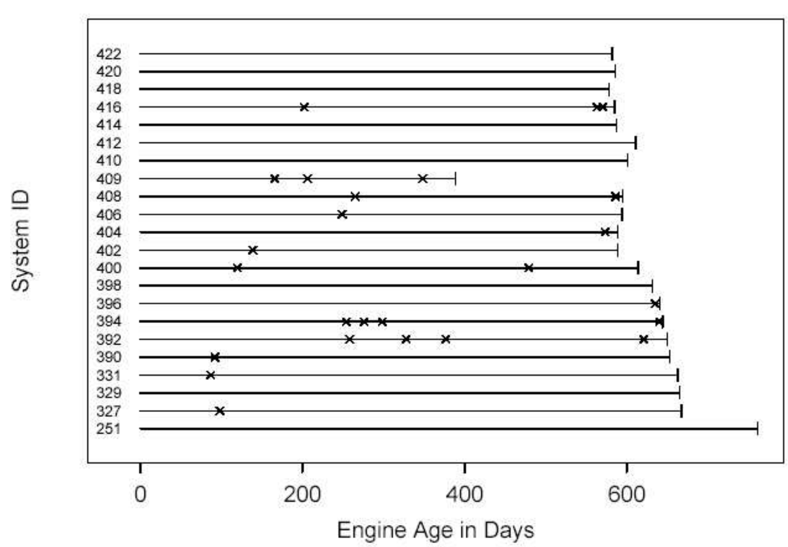
- **How to estimate the distribution of the failure time when there are censored observations?**
- **Probability of failure before 100 hours?**
- **Failure rate by 100 hours?**
- **Proportion failed after 10^5 hours?**

IC Data Failure Pattern



RECURRENT EVENTS/REPAIRABLE SYSTEMS

Valve Seat Replacement Times Event Plot
(Nelson and Doganaksoy 1989)

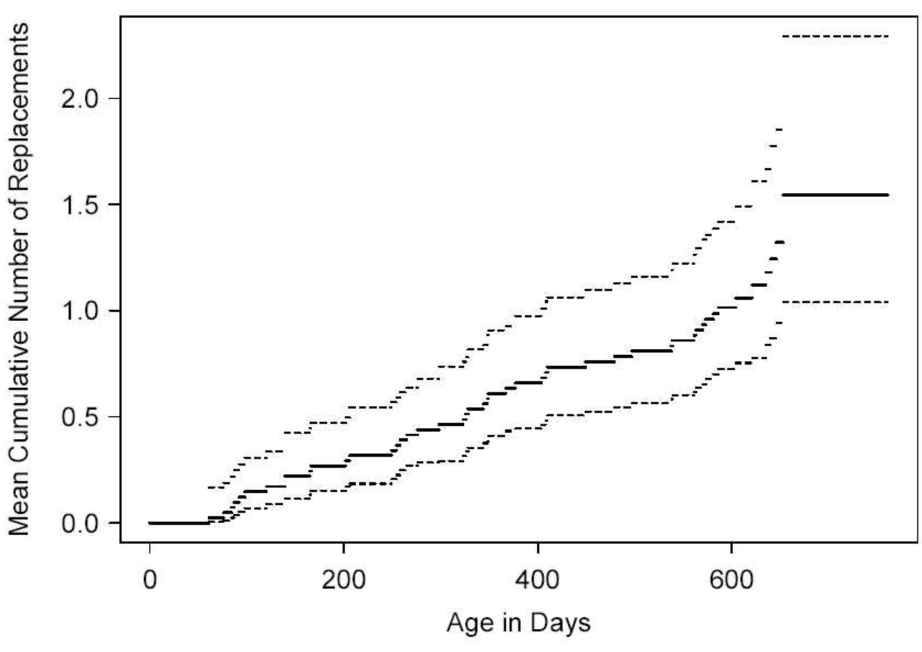


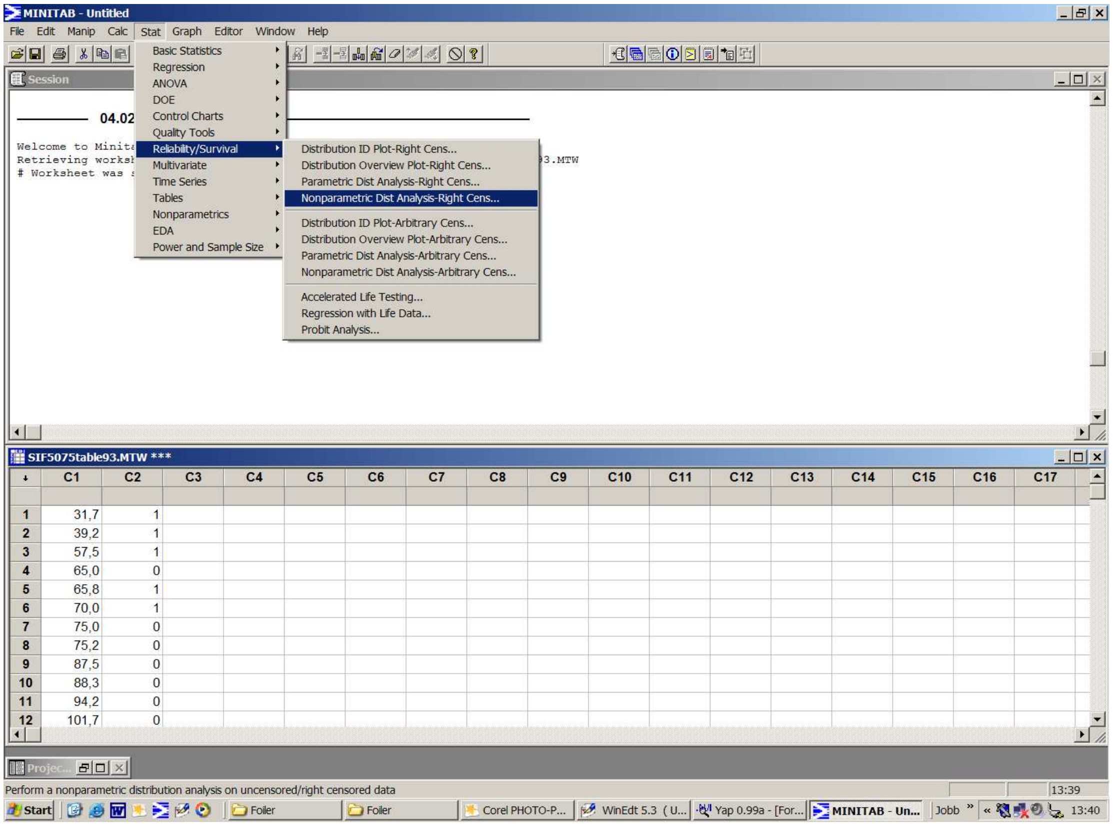
Valve Seat Replacement Times (Nelson and Doganaksoy 1989)

Data collected from valve seats from a fleet of 41 diesel engines (days of operation)

- Each engine has 16 valves
- Does the replacement rate increase with age?
- How many replacement valves will be needed in the future?
- Can valve life in these systems be modeled as a renewal process?

Estimate of Number of Valve Seat $\mu(t)$




MINITAB - Untitled
File Edit Manip Calc Stat Graph Editor Window Help
Session
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Welcome to Minit
Retrieving works
Worksheet was
Basic Statistics
Regression
ANOVA
DOE
Control Charts
Quality Tools
Reliability/Survival
Multivariate
Time Series
Tables
Nonparametrics
EDA
Power and Sample Size
Distribution ID Plot-Right Cens...
Distribution Overview Plot-Right Cens...
Parametric Dist Analysis-Right Cens...
Nonparametric Dist Analysis-Right Cens...
Distribution ID Plot-Arbitrary Cens...
Distribution Overview Plot-Arbitrary Cens...
Parametric Dist Analysis-Arbitrary Cens...
Nonparametric Dist Analysis-Arbitrary Cens...
Accelerated Life Testing...
Regression with Life Data...
Probit Analysis...
SIF5075table93.MTW ***
C1 C2 C3 C4 C5 C6 C7 C8 C9 C10 C11 C12 C13 C14 C15 C16 C17
1 31,7 1
2 39,2 1
3 57,5 1
4 65,0 0
5 65,8 1
6 70,0 1
7 75,0 0
8 75,2 0
9 87,5 0
10 88,3 0
11 94,2 0
12 101,7 0
Perform a nonparametric distribution analysis on uncensored/right censored data
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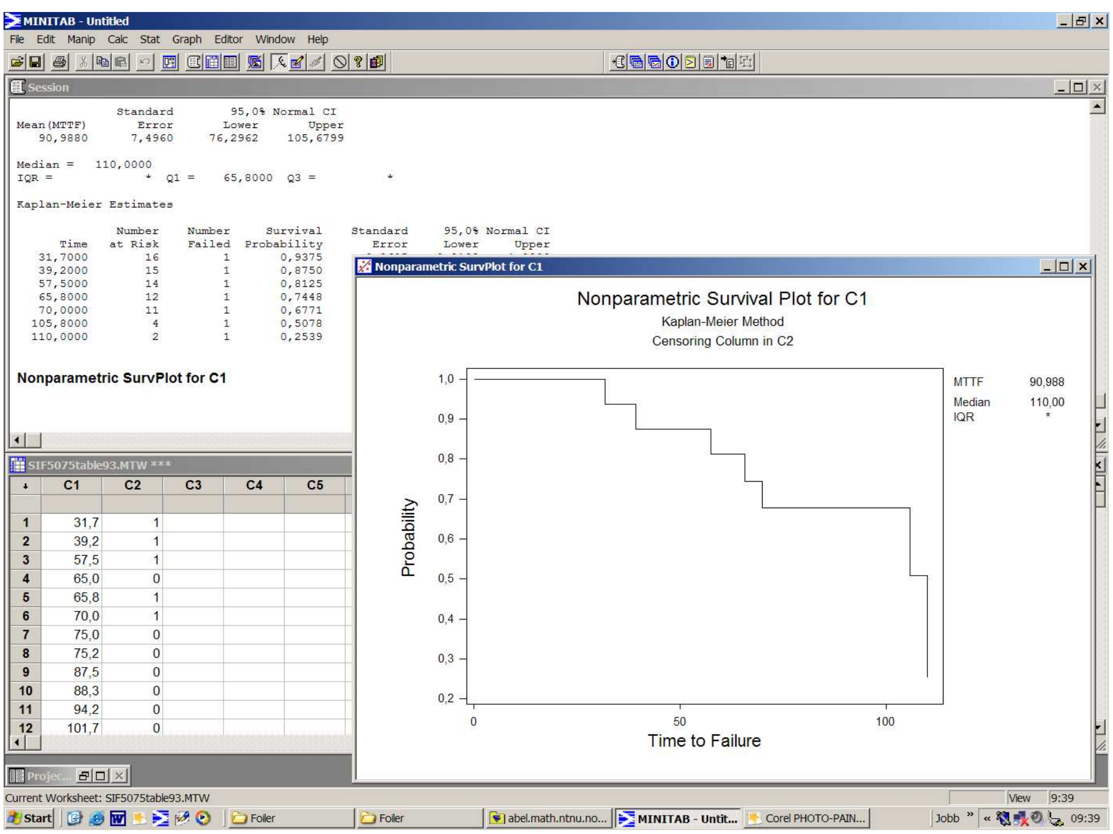
MINITAB - Untitled
File Edit Manip Calc Stat Graph Editor Window Help
Session
Standard 95,0% Normal CI
Mean(MTTF) Error Lower Upper
90,9880 7,4960 76,2962 105,6799
Median = 110,0000
IQR = * Q1 = 65,8000 Q3 = *
Kaplan-Meier Estimates
Number Number Survival Standard 95,0% Normal CI
Time at Risk Failed Probability Error Lower Upper
31,7000 16 1 0,9375
39,2000 15 1 0,8750
57,5000 14 1 0,8125
65,8000 12 1 0,7448
70,0000 11 1 0,6771
105,8000 4 1 0,5078
110,0000 2 1 0,2539
Nonparametric SurvPlot for C1
SIF5075table93.MTW ***
C1 C2 C3 C4 C5
1 31,7 1
2 39,2 1
3 57,5 1
4 65,0 0
5 65,8 1
6 70,0 1
7 75,0 0
8 75,2 0
9 87,5 0
10 88,3 0
11 94,2 0
12 101,7 0
Nonparametric SurvPlot for C1
Nonparametric Survival Plot for C1
Kaplan-Meier Method
Censoring Column in C2
MTTF 90,988
Median 110,00
IQR *
1,0
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
Probability
0
50
100
Time to Failure
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