TMA 4275 Lifetime Analysis 2018
Homework 3

Problem 1

Let the observed life times (no censorings) for 12 identical components be:

10.2, 89.6, 54.0, 96.0, 23.3, 30.4, 41.2, 0.8, 73.2, 3.6, 28.0, 31.6

Analyse these data fitting both parametric models such as the Weibull, the lognormal and the exponential and non-paramterically by computing the Kaplan-Meier estimate of R(t).

- R: First load the survival package by doing `library(survival)`, then create an object, say, `data` containing the data using the `c` and `Surv` functions, and fit parametric models, using for example `survreg(data ~ 1, dist="Weibull")`. Add plots of parametric estimates of the survival function (using `curve` and e.g. `pweibull`) to plots of the Kaplan-Meier non-parametric estimate of $R(t)$ obtained with `plot(survfit(data 1))`. Note that `survreg` parameterizes the Weibull based on its log-location-scale representation, that is, $\mu = \ln\theta$ and $\sigma = 1/\alpha$, rather than in terms of the more usual scale and shape parameters θ and α (used by `pweibull` and related functions).
- Minitab:

  First use Stat > Basic Statistics to obtain summary statistics and standard plots for the data.

  Then use Stat > Reliability/Survival > Distribution Analysis (Right Censoring) to do some parametric and non-parametric analyses (Kaplan-Meier, i.e. empirical distribution function) of the data.

Problem 2

Load the Repair Times data from the course web page.

- R: Load the data by doing

```
time <- Surv(scan(
"https://www.math.ntnu.no/emner/TMA4275/2017v/datasett/repair.times.txt",dec=",",skip
```

- Minitab: You may either upload the Minitab worksheet (.MTW) which will automatically start Minitab, or copy the .txt file into the Minitab worksheet after you have opened Minitab. (Try both ways!)

These data are 90 repair times for a certain system and can be treated as a complete (i.e. non-censored) dataset of lifetimes.

- R: Plot the empirical distribution function with `plot(ecdf(time))`
- Minitab: Plot the *empirical distribution function* using Graph > Empirical CDF.

What is the difference between this plot and the *empirical survival function* that you would get by using the Kaplan-Meier method with no censored observations?

It is argued that a log-normal distribution fits the data well (and fits repair data in general).

- R: Check this using the same R functions as in problem 1.
- Minitab: Check this for the present data by plots and analyses using Minitab.

Also try the Weibull distribution. What is the conclusion?

Problem 3

a) Let T be exponentially distributed with failure rate λ, and let V be exponentially distributed with failure rate μ. Assume that T and V are stochastically independent. Let $Z = \min(T, V)$.

Show that Z is exponentially distributed with failure rate $\lambda + \mu$.

b) Consider a serial system with two components, A and B, with survival times T and V as above. What is the survival function $R(t)$ of this system? (Recall that a series system works as long as both components work).

Assume that the system fails. What is the probability that component A is the failure cause?

c) Consider now instead a parallel system with the two components A and B with lifetime distributions as above. What is now the survival function $R(t)$ of the system? (Recall that a parallel system works as long as at least one of the components works).

d) Suppose now that component A is used alone until it fails, and then component B is activated. The system is assumed to fail when B fails. Express the lifetime of this system as a function of T and V.

What is the MTTF of this system?

What is now the survival function $R(t)$ of the system?