# TMA4275 Lifetime Analysis 2017
# Exercise 6

## Problem 1 – The two-parameter exponential distribution

The two-parameter exponential distribution has density

$$f(t;\theta,\gamma) = \frac{1}{\theta}\exp\left\{-\frac{t-\gamma}{\theta}\right\} \text{ for } t \geq \gamma$$

where $\theta > 0$, $\gamma \geq 0$.

Assume that we have a right censored sample $(y_i, \delta_i)$, $i = 1, \ldots, n$ from this distribution. Assume that all censorings take place at or after time $\gamma$.

**a)** Find the log-likelihood function $l(\theta, \gamma)$ for these data.

**b)** Let $(\hat{\theta}, \hat{\gamma})$ be the maximum likelihood estimators of $(\theta, \gamma)$. Why do we have

$$\hat{\gamma} \leq y_{(1)}$$

where $y_{(1)}$ is the smallest time among $y_1, \ldots, y_n$?

Then find explicit expressions for $(\hat{\theta}, \hat{\gamma})$. Show in particular that we always have $\hat{\gamma} = y_{(1)}$

**c)** In the lectures we have considered a likelihood method for constructing confidence intervals for one of two parameters in a model. The method uses the following:

Let $\hat{\gamma}(\theta)$ be the MLE of $\gamma$ when $\theta$ is given. Then

$$W(\theta) = 2(\ell(\hat{\theta}, \hat{\gamma}) - \ell(\theta, \hat{\gamma}(\theta)))$$

is approximately $\chi^2_1$ when $\theta$ is the true parameter. (Note that $\tilde{\ell}(\theta) = \ell(\theta, \hat{\gamma}(\theta))$ is the so-called profile log likelihood of $\theta$).

Explain how this can be used to construct a confidence interval for $\theta$. Do the calculations of the interval as far as you get.

**d)** Use MINITAB to estimate the parameters when the Pike cancer data (see page 25 of Slides 10 from lectures) are assumed to follow a two-parameter exponential distribution.

**e)** Reconsider the assumption in the beginning of the Problem, that all censorings take place at or after time $\gamma$. Can you think of cases where censorings also before time $\gamma$ are possible? In such cases, how should the analysis in b) be modified?

## Problem 2 – Censoring and truncation

$n = 10$ units with exponentially distributed life times and MTTF$= \theta$ are put on test. At time $c = 10$ the test is ended (type I censoring), and $r = 4$ units have failed by that time. The observed lifetimes are

$$0.9,\ 2.8,\ 5.9,\ 7.4$$

**a)** Write down the likelihood function and compute the MLE for $\theta$. Which are the assumptions behind this approach?

**b)** Assume now that at the end of the experiment ($c = 10$) one does not know how many units were put on test, but only knows that the experiment has gone for 10 time units, with $r = 4$ failures at the times given.

How can you write down a likelihood for this case? (Hint: This is right truncation, see page 3 of Slides 11 from lectures).

Which are the assumptions behind this likelihood?

**c)** Maximize the likelihood in (b) to find the MLE under the conditions given there.

## Problem 3 – Weibull regression

Do Problem 1 b,c in Exam 2013V.