Norges teknisk–
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Contact during exam:
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EXAM IN TMA4275 LIFETIME ANALYSIS

Thursday 26 May 2011

Time: 09:00–13:00

Permitted Aids:

Statistiske tabeller og formler (Tapir forlag)
Approved Calculator (HP30S)
K. Rottmann: Matematisk formelsamling
One yellow paper (A4 with stamp) with your own formulae and notes

Grading: 15 June 2011

ENGLISH

Problem 1

Listed below are the times of remission (T in months) (freedom from symptoms in a precisely defined sense) of leukaemia patients treated with 6-P drug. Right censored observations are marked with an asterisk(*).
6^*, 6, 6, 6, 7, 9^*, 10^*, 10, 11^*, 13, 16, 17^*, 19^*, 20^*, 22, 23, 25^*, 32^*, 32^*, 34^*, 35^*

a) Calculate the Kaplan-Meier estimate for the survival probability R(23) based on the above data. Find an approximate 95% confidence interval for R(23).

b) Plot the Nelson-Aalen estimate of the cumulative hazard rate $Z(t)$ for the above data. Does this plot indicate that T is exponentially distributed? By using a relation between the reliability function $R(t)$ and the cumulative hazard function $Z(t)$, calculate a new estimate of $R(23)$. Compare it to the value obtained in (a) for $R(23)$.

c) The Kaplan-Meier estimate is a step function with jumps. Discuss the mechanism of jumps, i.e. when there are jumps and when there are no jumps (not for the above data, but in general). Comment about the sizes of these jumps.

Problem 2

Consider a nonhomogeneous Poisson procss (NHPP) $\{N(t), t \geq 0\}$ with rate (ROCOF):

$$w(t) = \begin{cases} 6 - 2t & \text{for} \quad 0 \leq t \leq 2 \\ 2 & \text{for} \quad 2 < t \leq 20 \\ -18 + t & \text{for} \quad t > 20 \end{cases}$$

a) Expalin briefly the main characteristics of a NHPP.
Make a sketch of $w(t)$ as a function of t. Using the plotted graph, comment about the behaviour of the process.

b) Find the expected number of failures in the time interval $(0, 24)$.
Also find the expected number of failures in the time intervals $(0, 12)$ and $(12, 24)$.

c) What is the probability that there will be exactly 1 failure in first 2 time units?
Given that the first failure happens at time $t = 1$, what is the probability that the system will not fail during the next 2 time units after the failure?

Problem 3

Below are given failure times (in days) of two specific machines, which are in constant use after being installed. Machine1 was observed for the first 25 days and machine 2 was observed for the first 30 days. It is assumed that the repairs are minimal and take negligible time, so that failure times of the two machines are assumed to follow nonhomogeneous Poisson processes (NHPP). Further the failures corresponding to the two machines are assumed to be independent.

Machine 1 :	10	18	21	
Machine 2 :	19	20	24	27

Our aim is to study the failure trends of the machines. For this,

a) Consider the Laplace trend test for a single system. Specify the null, and alternative hypothesis, the test statistic and explain the idea behind the test.

b) Perform a pooled Laplace test to investigate whether there is some trend in the failure patterns of the machines. Write the conclusion if a 5% level of significance is used.

c) Test whether the failure pattern of the machines have a trend using the TTT based Laplace trend test. Again use a 5% level of significance. What is the difference between the hypotheses tested in b) and c), respectively?

Problem 4

In some applications a third parameter, called a guarantee time, is included in the models like for example the Weibull model. This parameter $\phi \geq 0$ is the smallest time at which a failure could occur. The survival (reliability) function of the three-parameter Weibull distribution is given by

$$S(x) = \begin{cases} 1 & \text{if} \quad x < \phi \\ \exp[-\lambda(x-\phi)^{\alpha}] & \text{if} \quad x \geq \phi \end{cases}$$

where $\alpha > 0$

a) Find the hazard rate and the density function of the three-parameter Weibull distribution.

b) Suppose that the survival time X follows a three-parameter Weibull distribution with $\alpha = 1$, $\lambda = 0.0075$ and $\phi = 100$.
Find the mean lifetime.
Find the mean residual lifetime at time 200.

c) Consider the model

$$Y = \ln X = \mu + \sigma W$$

for a lifetime X. Assume that W is distributed with the following density function

$$f_W(w) = e^{w} e^{-e^{w}} \qquad \text{for} \quad -\infty < w < \infty$$

Show that X is Weibull distributed. What are the parameters of this distribution?