TMA4285 Time series models
Project 1, Problem 1
Autumn 2015

Consider the ARMA(2,1) model

$$(1 - \phi_1 B - \phi_2 B^2)(Z_t - \mu) = (1 - \theta_1 B)a_t,$$

where $\{a_t\}_{t=-\infty}^{\infty}$ is Gaussian distributed white noise with zero mean and $\mathrm{Var}[a_t] = \sigma_a^2$.

a) For the above model, find analytical formulas for the variance $\mathrm{Var}Z_t$ and the correlation function $\rho_k, k = 1, 2, \ldots$. (It is sufficient that you find explicit formulas the variance and for ρ_1 and ρ_2, and a recursive formula that can be used to compute ρ_k for $k = 3, 4, \ldots$.)

b) Using the R function arima.sim, simulate $\{z_t\}_{t=1}^{200}$ from the above model when $\phi_1 = 0.2$, $\phi_2 = 0.5$, $\theta_1 = -0.8$, $\mu = 1$ and $\sigma_a^2 = 0.25$. Note that arima.sim is based on a parameterization of the MA-part of the model slightly different from that of Wei. Plot the realisation.

Use the simulated realisation to estimate the acf and the pacf of the model for lags up to $k = 50$. To study the bias and variance of the sample acf at different lags k, repeat the process of simulation and estimation several times. Visualise your results and use them to check your analytical results in **a**).

From the simulated realisations of the sample acf, also estimate the correlation between $\hat{\rho}_i$ and $\hat{\rho}_j$ for all combinations of i and $j = 1, 2, \ldots, 50$. Visualise the resulting correlation matrix in a suitable way and briefly discuss the result.

c) For the model parameters used in **b**) compute numerical values for $\phi_{kk}, k = 1, 2, \ldots, 50$ by implementing a function which computes the pacf from the acf using eqs. (2.5.25) and (2.5.26) in Wei.

Estimate ϕ_{kk} from a realisation from the model. Again study the estimation uncertainty and bias by repeating the process of simulation and estimation.

Visualise again your results and use them to check your analytical results.

d) You should now make your own function which can simulate from the above model. Your function does not need to be general, it is sufficient that it can simulate from the above model with parameter values as specified in **b**). You should only use the function rnorm to generate random numbers. Thus, you are not allowed to use the arima.sim function or any other random generating functions, except rnorm. In addition to including the code in your solution, you should also explain your simulation strategy in normal text and include analytical calculations that is necessary to understand your simulation strategy. In particular, carefully consider how to choose initial values.

Check your implementation by repeating the simulation and estimation parts of **b**) and **c**) using your own function in stead of arima.sim.