Norges teknisk–
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EXAM IN TMA4285 TIME SERIES MODELS

Monday 04 December 2006
Time: 09:00–13:00

Tillatte hjelpemidler:
Tabeller og formler i statistikk, Tapir Forlag
K.Rottmann: Matematisk formelsamling
Bestemt, enkel kalkulator tillat
Ett gult A4-ark med stempel med egne formler og notater

Sensur: 24 December 2006

Oppgave 1

Let X_t be the ARMA(1,1) process defined by

$$X_t - \theta X_{t-1} = Z_t + \theta Z_{t-1}, \tag{1}$$

where $|\theta| < 1$, $Z_t \sim \text{WN}(0, 1)$.

a) Give definitions of causality and invertibility. Is this process causal? Invertible? Why?

b) Find coefficients ψ_j, $j = 0, 1, \ldots$ of the representation

$$X_t = \sum_{j=0}^{\infty} \psi_j Z_{t-j}.$$

(Hint: write out operator $(1 - \theta B)^{-1}$ and apply it to both sides of (1))

c) Find the ACVF of X_t as follows. First obtain equations from which $\gamma(0)$ and $\gamma(1)$ can be found. Then for $k \geq 2$ express $\gamma(k)$ in terms of $\gamma(k-1)$.

Oppgave 2

Let X_t be the AR(2) time series defined by

$$X_t - \phi(1+\phi)X_{t-1} + \phi^3 X_{t-2} = Z_t,$$

where $|\phi| < 1$, $Z_t \sim \mathrm{WN}(0, \sigma^2)$.

a) Prove that this process is causal.

b) Let $n > 1$. Find $P_n X_{n+1}$ the best linear predictor of X_{n+1} in terms of $X_1, ..., X_n$.

Consider the process Y_t defined by

$$Y_t = \left(1 - \frac{1}{2}B^2\right) X_t.$$

c) Show that Y_t is stationary.

d) Let $n > 1$. Find $P(Y_{n+1}|X_1, ..., X_n)$ the best linear predictor of Y_{n+1} in terms of $X_1, ..., X_n$.

Oppgave 3

Establish which of the following two functions is the autocovariance function of a stationary process and which is not:

$$\gamma_1(h) = \begin{cases} 1 & \text{if } h = 0, \\ 1/2 & \text{if } h = \pm 1, \\ 2/3 & \text{if } h = \pm 2, \\ 0 & \text{otherwise.} \end{cases} \qquad \gamma_2(h) = \begin{cases} 0.4 & \text{if } h = 0, \\ -0.1 & \text{if } h = \pm 5, \\ -0.1 & \text{if } h = \pm 7, \\ 0 & \text{otherwise.} \end{cases}$$