Norwegian University of Science and Technology
Department of Mathematical Sciences

English

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EXAM IN TMA4285 TIME SERIES MODELS
9. December 2009
Time: 09:00-13:00

Permitted aids: Tabeller og formler i statistikk, Tapir Forlag
K. Rottman: Matematisk formelsamling
Calculator HP30S or CITIZEN SR-270X
One yellow, stamped A5 sheet with own formulas and notes.
The results of the exam are available January 11. 2010.

Problem 1

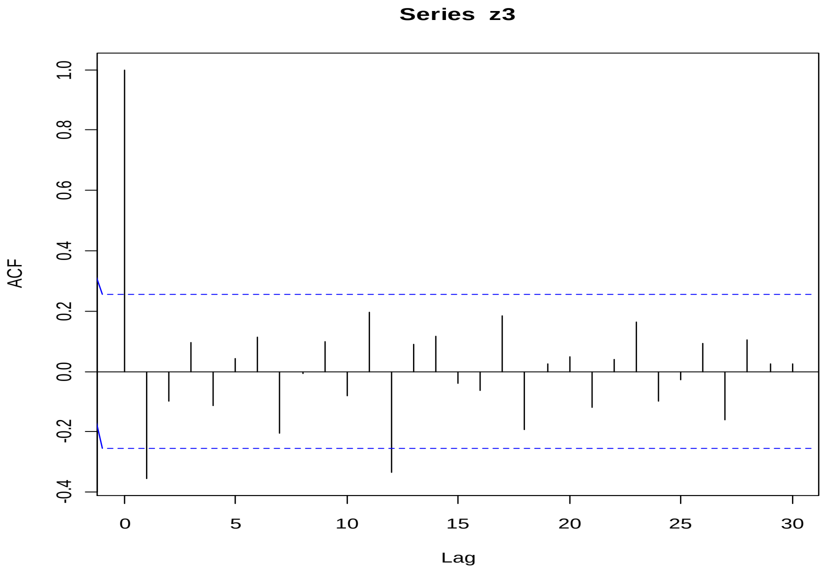
a) What properties characterize a second order weakly stationary process?
Is the process given by

$$Z_t = (1-\theta_1 B)(1-\theta_{12}B^{12})a_t$$

where the sequence $\{a_t\}$ is white noise, a second order weakly stationary process? Explain your answer.

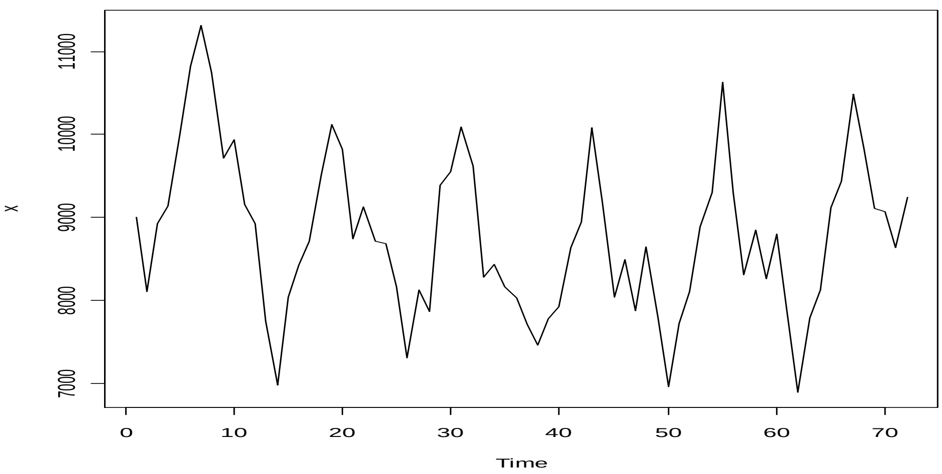
b) What are the requirements for the process given in 1a) to be invertible?
Find an expression for the autocovariance function for $\{Z_t\}$.

c) Assume $\theta_1 = 0.4$ and $\theta_{12} = 0.5$. Calculate the autocorrelation function for $\{Z_t\}$ for these values of θ_1 and θ_{12}.
A plot of the sample autocorrelation function for a time series is given below. Discuss whether this sample autocorrelation function can be generated from the process (time series) given in 1a)



d) Assume the process given in 1a) is invertible. Then we know that $a_t = \pi(B) Z_t$.
Find the $\pi -$weights and explain how you can construct one step and two steps ahead forecasts with such a model. Which forecasts will be zero.

The time series given below represents the monthly accidental deaths in U.S.A. from 1973 to 1978, starting in January 1973 and ending in December 1978.



e) Denote the time series by $\{X_t\}$. A plot of the sample autocorrelation function of $(1-B)(1-B^{12})X_t$ is given in 1c). Suggest a model for the time series $\{X_t\}$.
The time series shows that the minimum number of accidental deaths always happens in February each year. Show how forecasts for February 1979 can be generated. You can use results from 1d). Assume normal distributed data and show also how a 95% prediction interval for the number of accidental deaths in February 1979 can be constructed.

Problem 2

A second order weakly stationary ARMA process is given by:

$$(1-\phi_2 B^2)Y_t = (1-\theta_1 B)a_t$$

where the sequence $\{a_t\}$ is white noise.

a) Find the autocovariance function for $\{Y_t\}$.
Show that the autocorrelation at lag one is always less than $1/2$ in absolute value if $\phi_2 \le 0$.

b) Give a state space representation of the model for $\{Y_t\}$

c) Develop the general expression for the forcast function for the process $\{Y_t\}$.
Assume that $a_t = \sigma_t e_t$ where the sequence $\{e_t\}$ consists of i.i.d random variables with mean 0 and variance 1 satisfying that e_t is always independent of a_{t-k} for $k > 0$. Assume further that $\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2$. Develop formulas for the one step ahead and the two steps ahead conditional forecast error variances.