Norwegian University of Science and Technology
Department of Mathematical Sciences

English

Contact person during the exam:
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EXAM IN TMA4285 TIME SERIES MODELS
8. December 2010
Time: 09:00-13:00

Permitted aids: Tabeller og formler i statistikk, Tapir Forlag
K. Rottman: Matematisk formelsamling
Calculator HP30S or CITIZEN SR-270X
One yellow, stamped A5 sheet with own formulas and notes.
The results of the exam are available January 11. 2010.

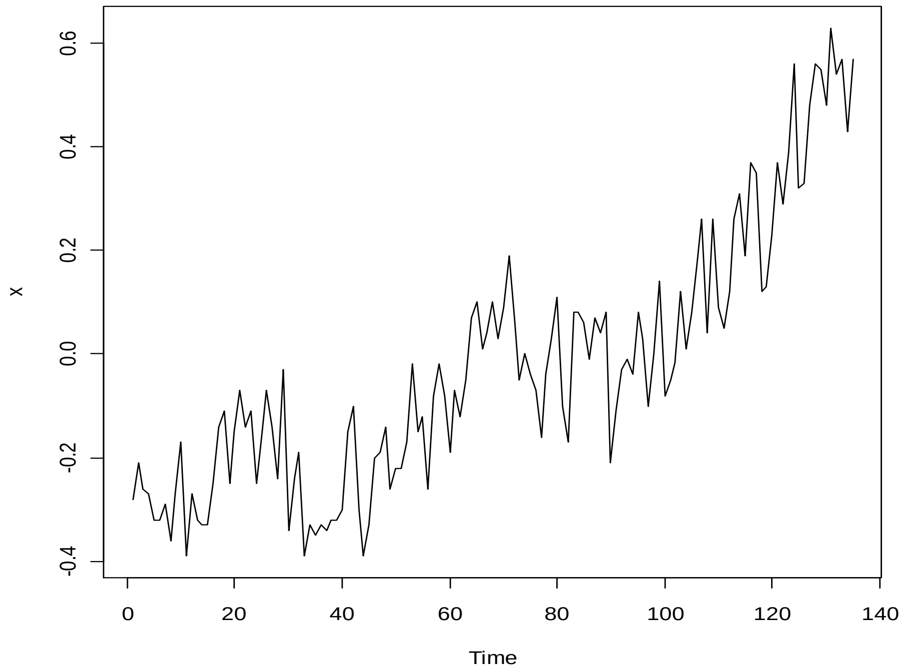
**Problem 1**

a) What are the normal estimators for the mean, the variance and the autocorrelation function in a second order weakly stationary process?

b) Is the process given by $Z_t = \left(1 - 0.5B - 0.14B^2\right) a_t$, where the sequence $\{a_t\}$ is white noise, second order weakly stationary? Is it invertible?
Suppose we want to fit an AR(p) process to observations from such a process. Which model is likely to be chosen. Explain your answers.

**Problem 2**

The figures below shows 130 yearly average global temperatures from 1880-2009 given as deviations from the average from 1961 to 1990, and estimated autocorrelations and partial autocorrelations for the same time series.

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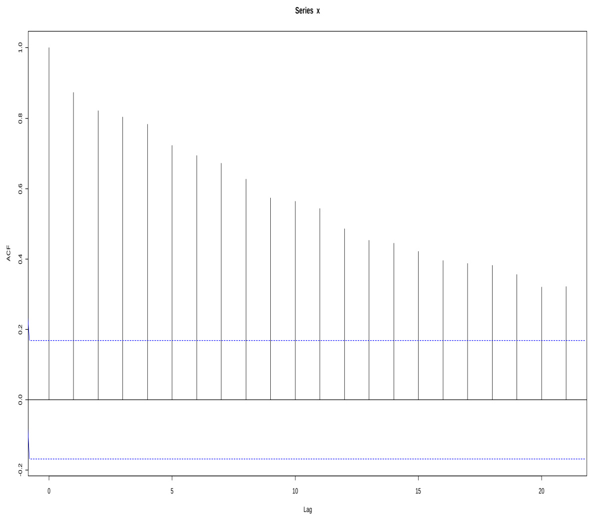


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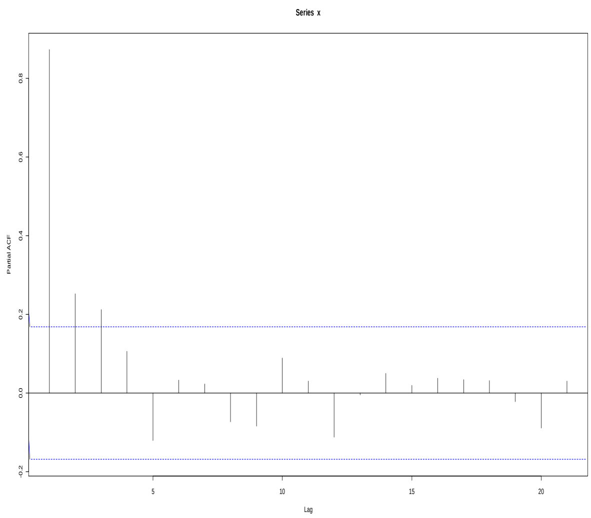


a) Let $\{X_t\}_{t=1}^{130}$ denote the stochastic process that has generated the time series above. Would you suggest to difference the time series? Explain your answer.
Show that the model

$$(1-B)X_t = \theta_0 + a_t,\ \theta_0 \neq 0.$$

where the sequence $\{a_t\}$ is white noise, has a linear trend.

b) A possible model for $X_t$ is

$$(1-B)X_t = (1-\theta_1 B)a_t + \theta_0$$

Let $Y_t = (1-B)X_t = (1-\theta_1 B)a_t + \theta_0$. Find the auto-covariance function for $Y_t$.

Show that the variance of $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$ is given by $\frac{\sigma_a^2}{n}\left((\theta_1 - 1)^2 + \frac{2\theta_1}{n}\right)$ where $n$ is the number of observations for $Y_t$.

c) What is a natural estimator for $\theta_0$?
Estimated values of the parameters for the model in 2b) are:
$\hat{\theta}_0 = 0.0066$, $\hat{\theta}_1 = 0.6$ and $\hat{\sigma}_a^2 = 0.001$.
Test if $\theta_0$ is significant different from zero with this model. Use the result from 2b.
Would you conclude that there has been a global warming over these 130 years?

d) The last observation for $X_t$, $x_{130} = 0.57$ and the last residual, $\hat{a}_{130} = 0.07$.
The average of the average global temperature from 1961-1990 is $14.5^0\,C$ (see introduction to problem 2). Predict the average global temperature in 2010.
Find the predicted average global temperature in 2100. Assume normally distributed data and construct a 95 % prediction interval for the average global temperature in 2100.

e) It turns out that the model $(1-B)X_t = (1-0.5B-0.14B^2)a_t$ gives a slightly better fit to the data. Do you believe that the model $(1-\phi_1 B-\phi_2 B^2-\phi_3 B^3)(1-B)X_t = a_t$ with suitable values for $\phi_1, \phi_2$ and $\phi_3$ also could be an alternative model for the data (look back to 1b)). Evaluate if the uncertainty in the predicted average global temperature for 2100 would be larger or smaller with the model $(1-B)X_t = (1-0.5B-0.14B^2)a_t$ than with the model in 2b) and 2d).

f) The linear trend in the plot does not seem to be constant over time and some scientists argue that they rather believe in a model with varying linear trend, i.e. a model of the type:

$$X_t = X_{t-1} + \beta_{t-1} + a_t$$
$$\beta_t = \beta_{t-1} + e_t$$

where $\{a_t\}$ and $\{e_t\}$ are independent white noise processes.
Write this model as a state space model.
Assume $\hat{\beta}_t$ for $t = 2009$, equals 0.0066. Use the state space model to also predict the average global temperature in 2010 and 2100.