Norwegian University of Science and Technology
Department of Mathematical Sciences

English

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EXAM IN TMA4285 TIME SERIES MODELS
December 1. 2011
Time: 09:00-13:00

Permitted aids: Tabeller og formler i statistikk, Tapir Forlag
K. Rottman: Matematisk formelsamling
Calculator HP30S or CITIZEN SR-270X
One yellow, stamped A5 sheet with own formulas and notes.
The results of the exam are available December 22. 2011.

Problem 1

a) Two stochastic processes are given by:

1 $\quad (1-\phi B)Z_t = (1-\theta B)a_t$

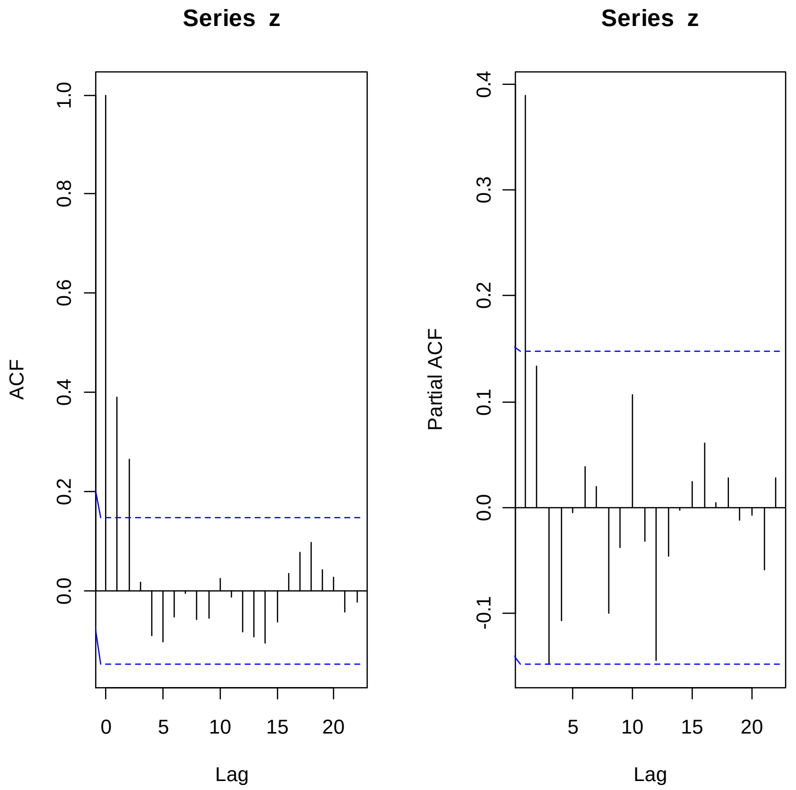
2 $\quad Z_t = (1-\theta_1 B - \theta_2 B^2)a_t$

where the sequence $\{a_t\}$ is white noise. When are the two time series processes second order weakly stationary? Find the autocorrelation function for the second process (the MA(2) process).

b) Assume that process 1 is weakly stationary. Show that the autocorrelation at lag one is given by:

$$\rho_1 = \frac{(\phi-\theta)(1-\phi\theta)}{1+\theta^2-2\phi\theta}$$

Find an expression for the autocorrelation at lag k, ρ_k.

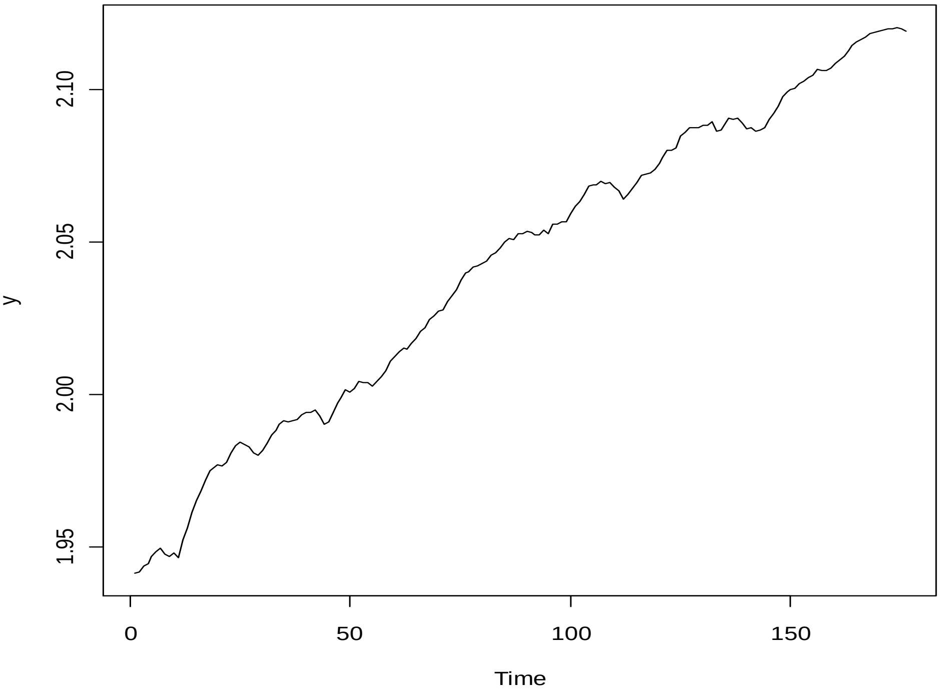


c) A plot of the estimated autocorrelation function and the estimated partial autocorrelation function of a time series is given above. The estimated partial autocorrelation function has one significant term at lag 1. Suppose you have the choice between an AR(1) or an ARMA(1,1) model for this time series. Which one would you prefer based on these plots? Explain your answer. Can the estimated autocorrelation function support a MA(2) process? (see problem 1a).

d) Write the ARMA(1,1) model $(1-\phi B)Z_t = (1-\theta B)a_t$ as an infinite moving average model and determine the weights.
An ARMA(1,1) model fitted to the data gave $\phi_1 = 0.8$ and $\theta_1 = 0.34$. Is it likely that such a model can be approximated well with an MA(2) model.

A plot of the actual time series, log transformed, is given on the next page. The time series represents 176 seasonal adjusted observations of the quarterly U.S. GNP in a suitable unit. The plot of the estimated autocorrelations and the partial autocorrelations given in 1c) are obtained from this time series after it has been differenced and log transformed.

Plot of the log transformed U.S. GNP

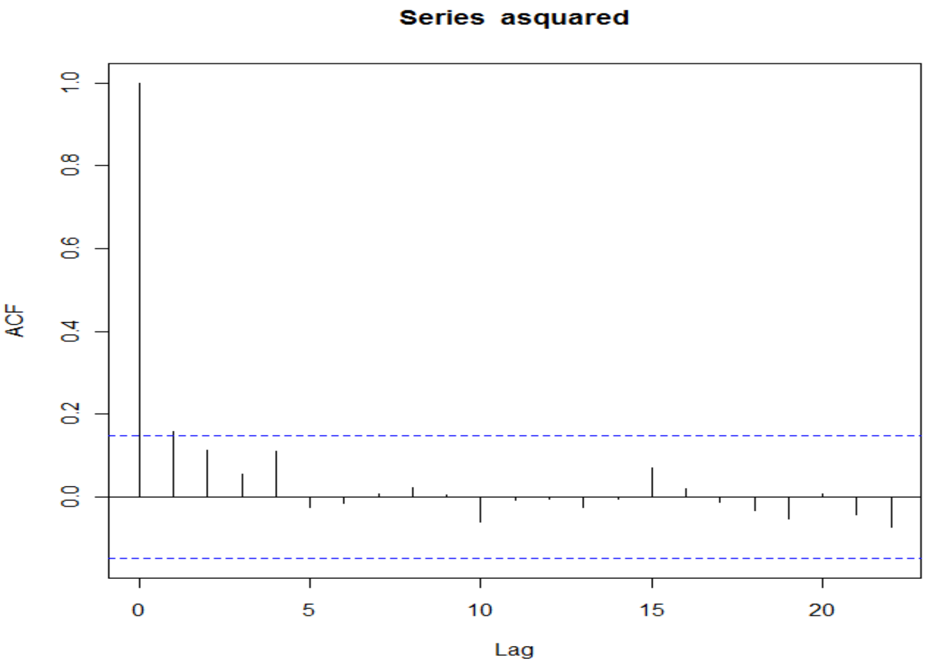


Let $\{Y_t\}_{t=1}^{176}$ be the sequence of stochastic variables that has generated the quarterly U.S. GNP data. It turns out that a well fitting model for the log transformed time series is given by: $(1-B)\ln(Y_t)=(1-\theta_1 B-\theta_2 B^2)a_t+\theta_0$. Estimated values for θ_1, θ_2, σ_a and θ_0 are -0.32,- 0.28, 1.3×10^{-3} and 0.001 respectively.

e) The last observation for $\ln\{Y_t\}$ is 2.1193 and the two last residuals are $\hat{a}_{176}=-0.00063$ and $\hat{a}_{175}=-0.00059$. Assume the sequence is $\{a_t\}\sim N(0,\sigma_a^2)$. Determine the one step ahead and the two step ahead forecasts for $\ln\{Y_t\}$ given all the observation up to time 176 and construct the two-step ahead 95% prediction interval for $\ln\{Y_t\}$.

The figure below shows a plot of the estimated autocorrelations of the squared residuals.

Plot of the estimated autocorrelation for $\hat{a}_t^2$



An estimated model for $\sigma_t^2 = E\left(a_t^2 | a_{t-1}, a_{t-2}, \ldots\right)$ is given by $\sigma_t^2 = 0.65\sigma_{t-1}^2 + 0.17a_{t-1}^2$

Let the white noise process $\{\varepsilon_t\}$ be defined by $\varepsilon_t = a_t^2 - \sigma_t^2$.

f) Show that a model for the squared residuals then will be given by:

$$a_t^2 - 0.82a_{t-1}^2 = \varepsilon_t - 0.65\varepsilon_{t-1}$$

Is this model reasonable consistent with the autocorrelation plot? (You may use result from 1b)) Construct an ARCH (2,0) approximation for this model. Use the ARCH(2) model to evaluate if the two step ahead 95% prediction interval in 1e) will be wider or narrower taking the conditional heteroscedasticity into account.

g) For x small in absolute value we have $\ln(1+x) \approx x$. In economics the rate of return, p_t, for an investment x_t is defined as $p_t = \frac{x_t - x_{t-1}}{x_{t-1}} = \frac{x_t}{x_{t-1}} - 1$. Explain why $X_t = \left(1 - \theta_1 B - \theta_2 B^2\right) a_t + \theta_0$ can be be a model for the rate of return for the seasonal adjusted U.S. GNP at time t. How will you interpret the parameter θ_0?

h) Let the two step ahead prediction intervals for $\{\ln(Y_t)\}$ be given by (L_l, L_u). What will then be the two step ahead prediction interval for $\{Y_t\}$?

Find the state space representation of the model $(1-B)\ln(Y_t) = \left(1 - \theta_1 B - \theta_2 B^2\right) a_t$.