Norwegian University of
Science and Technology

Department of Mathematical Sciences

Examination paper for **TMA4285 Time Series Models**

Academic contact during examination: Professor Gunnar Taraldsen

Phone: 46432506

Examination date: December 11, 2018

Examination time (from–to): 15:00 – 19:00

Permitted examination support material: C

— Tabeller og formler i statistikk, Akademika
— K.Rottman. Matematisk formelsamling
— Stamped yellow A5 sheet with your own handwritten notes
— Determined, single calculator

Other information:

You may write in English or Norwegian.

All answers have to be justified. They should include enough details in order to see how they have been obtained.

All 20 sub-problems carry the same weight for grading.

Language: English

Number of pages: 4

Number of pages enclosed: 0

Checked by:

Informasjon om trykking av eksamensoppgave	
Originalen er:	
1-sidig ☐	**2-sidig** ☒
sort/hvit ☐	**farger** ☒
skal ha flervalgskjema ☐	

Date Signature

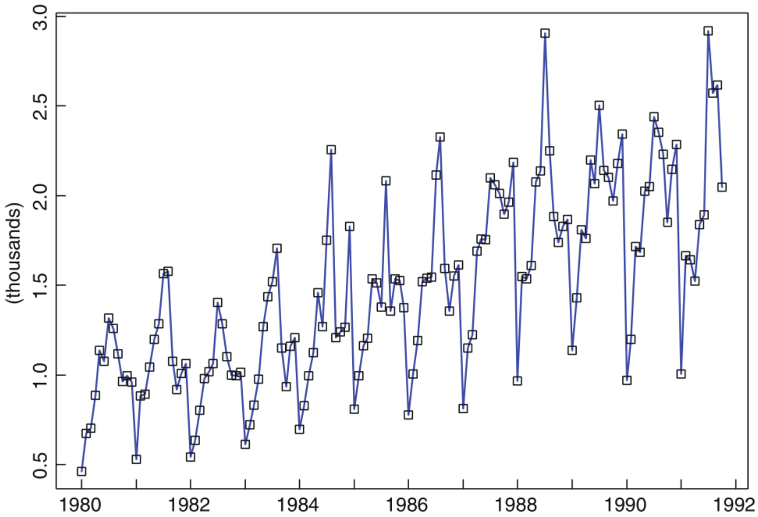

Figure 1: A time series.

Problem 1 Figure 1 shows the monthly sales in kiloliters of red wine by Australian winemakers from January 1980 through October 1991. In the following it will be assumed that the monthly sales is a realization of a stochastic process U_t.

a) What is the mathematical definition of a stochastic process?

b) Justify that the likelihood of the observations in Figure 1 can be written on the form

$$L(\theta) = \prod_{i=1}^{n} f_i(u_i \,|\, u_{i-1}, \ldots, u_1) \tag{1}$$

c) How does L simplify if the observations are independent?

d) How does L simplify if U_t is a Gaussian process?

e) How can L be used to estimate θ?

f) Explain why the conditional expectation $\mathrm{E}(U_{n+h} \,|\, U_1, \ldots, U_n)$ is better than the best linear predictor $\hat{\mathrm{E}}(U_{n+h} \,|\, U_1, \ldots, U_n)$ of U_{n+h}.

g) Derive a matrix formula for the best linear predictor.

h) Explain how, in principle, to forecast the wine sales in January 1992.

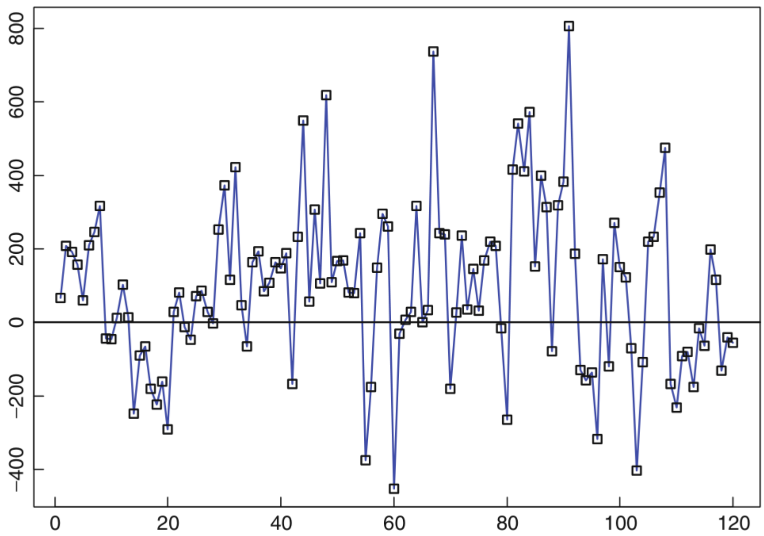

Figure 2: A transformed time series.

Problem 2 Figure 2 shows the time series $x_t + 0.0681 = (1 - B^{12})y_t$ where $y_t = \log(u_t)$ and u_t is the time series in Figure 1. In the following it will be assumed that X_t is a zero-mean weakly stationary time series.

a) Why is it reasonable to consider the time series $y_t = \log(u_t)$?

b) Why is it reasonable to consider the time series $(1 - B^{12})y_t$ obtained by differencing at lag 12?

c) Define mathematically what it means that X_t is an ARMA(p, q) process.

d) How can the likelihood of a Gaussian ARMA(p, q) process be computed?

Figure 3 shows the sample partial autocorrelation function of the data in Figure 2.

e) Figure 3 can be taken as an argument for assuming that X_t is an AR(12) process. Explain this.

f) Forecasting for an AR(p) process is particularly simple. Explain this.

g) It can be shown that the AR(12) process $(1 - 0.270B - 0.224B^5 - 0.149B^8 + 0.099B^{11} + 0.353B^{12})X_t = Z_t$ with $Z \sim \text{WN}(0, 0.0138)$ gives a good fit to the data. Explain how this can be used to forecast the wine sales in January 1992.

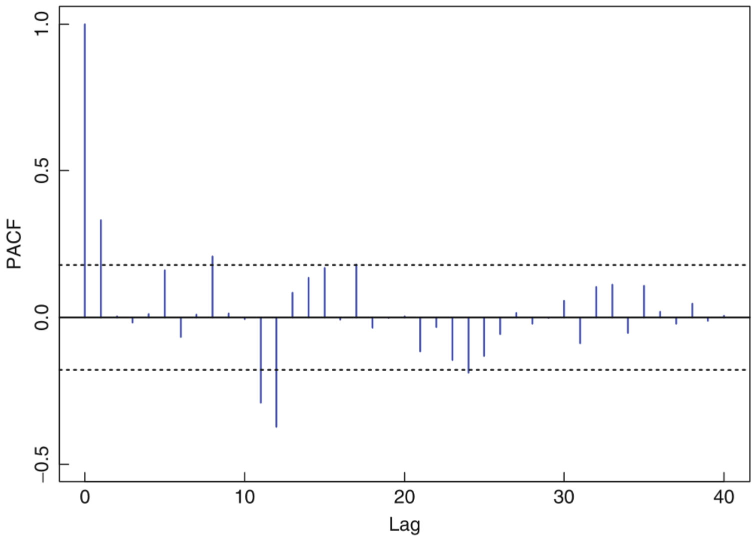

Figure 3: A sample partial autocorrelation function.

Problem 3 Figure 4 shows $Z_t = \log(P_t/P_{t-1})$ where P_t is the Dow Jones Industrial Index from July 1, 1997, through April 9, 1999 together with an estimate of h_t in a causal stationary GARCH$(1,1)$ model: $Z_t = \sqrt{h_t}e_t$, $h_t = \alpha_0 + \alpha_1 Z_{t-1}^2 + \beta_1 h_{t-1}$, and $e_t \sim \mathrm{IID}(0,1)$.

a) What does it mean that Z_t is causal and stationary?

b) Show that $\mathrm{E}(Z_t^2 \mid Z_{t-1}^2, Z_{t-2}^2, \ldots) = h_t$.

c) Show that $Z_t \sim \mathrm{WN}(0, \sigma_Z^2)$ including an explicit formula for σ_Z^2.

d) What is the interpretation of the volatility h_t?

e) GARCH models have been developed to reflect the so-called stylized features of financial time series. Exemplify some of these features.

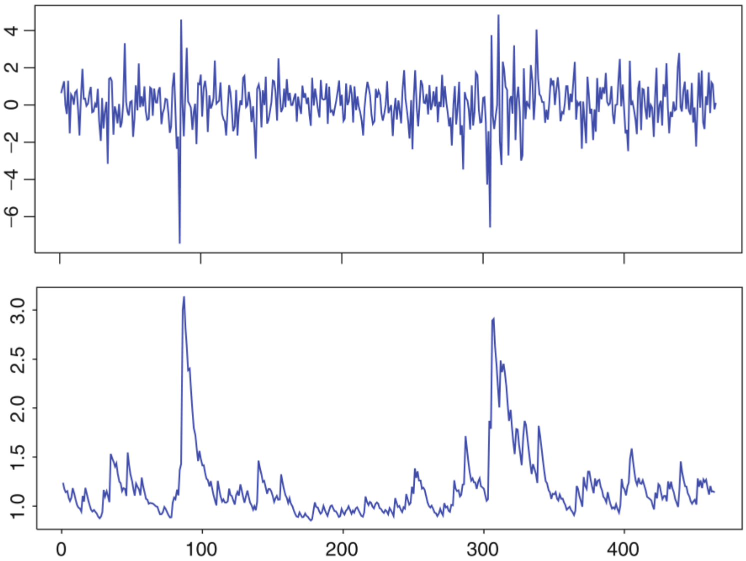

Figure 4: A white noise process (top) together with estimated volatility.