TMA 4295 Statistical Inference 2014
Homework 10

Problem 1

Let $X_1, \ldots, X_n$ be i.i.d. $N(\theta, \sigma_0^2)$, where σ_0^2 is known.

a) Find the strongest α-level test for

$$H_0 : \theta = \theta_0 \text{ vs } H_1 : \theta = \theta_1$$

1. if $\theta_1 > \theta_0$
2. if $\theta_1 < \theta_0$

Why do you get two different tests?

b) Find UMP tests with level α for

1. $H_0 : \theta \leq \theta_0$ vs $H_1 : \theta > \theta_0$
2. $H_0 : \theta \geq \theta_0$ vs $H_1 : \theta < \theta_0$

c) Assume we want to test

$$H_0 : \theta = \theta_0 \text{ vs } H_1 : \theta \neq \theta_0$$

The likelihood ratio test (LRT) for this problem consists in rejecting H_0 if

$$|\bar{X} - \theta_0| \geq \frac{\sigma_0}{\sqrt{n}} z_{\alpha/2}$$

Show that this test is *not* a UMP test.

d) Try to explain why there can be no UMP test for the hypothesis in c). See also Example 8.3.19 in the book. (It turns out, however, that the test in c) is uniformly best in the class of tests with power function which is never under α in the alternative ($\theta \neq \theta_0$). This is treated in Example 8.3.20 in the book and is not in our syllabus).

Problem 2

Do Exercise 8.33 on p. 407 in the book.

Problem 3

The following is taken from the beginning of an article in The American Mathematical Monthly, 1990. What is your opinion about Dr. Jones' interval? Is it really a 95% confidence interval?

A Remark on the Shortest Confidence Interval of a Normal Mean

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1. Introduction. In estimation theory, the shortest 95% confidence interval for the mean of a normal distribution of known variance σ^2 is given by

$$\bar{X} \pm 1.96\sigma/\sqrt{n} \tag{1}$$

Dr. Jones, a recent graduate from the *Department of Dubious Statistics*, found an even shorter confidence interval. First he computed the interval given in (1). He then pretended that σ is unknown and computed the interval

$$\bar{X} \pm tS/\sqrt{n-1} \tag{2}$$

where $nS^2 + \sum_{i=1}^{n}(X_i - \bar{X})^2$ and $t = t_{.975}^{(n-1)}$ is the 97.5th percentile of a student's t-distribution with $n-1$ degrees of freedom. Now he chooses whichever of intervals (1) and (2) that happened to be shorter.

Obviously, the length of Dr. Jones' interval is less than or equal to the length of inteval (1). On the other hand, both (1) and (2) have coverage probability of .95, so Dr. Jones claims that the same must be true for his interval.

Has Dr. Jones made a significant statistical discovery?