Exercise 12 TMA4295

Problem 1

Exercise 9.11 in the text book

Problem 2

Exercise 9.25 in the text book

Problem 3

A sample $X_1, ..., X_n$ is taken from a gamma distribution with parameters θ and

$1/\theta$:

$$X_1, ..., X_n \sim gamma\left(\theta, \frac{1}{\theta}\right)$$

i. e. pdf of X_i is

$$f(x; \theta) = \frac{\theta^\theta}{\Gamma(\theta)} x^{\theta-1} e^{-\theta x} I_{\{x>0\}}, \quad \theta > 0.$$

Find a one-dimensional sufficient statistic for θ.

Problem 4

Let $X_1, ..., X_n$ be a sample taken from a normal distribution with zero mean and

unknown variance θ^2:

$$X_1, ..., X_n \sim N(0, \theta^2)$$

a) Find the (expected) Fisher information.

b) Consider the following estimator of θ^2:

$$T_n = \frac{2}{n}X_1^2 + \frac{n-2}{n(n-1)}\sum_{i=2}^{n} X_i^2.$$

Is this estimator unbiased?

c) Is T_n consistent?

d) Is the estimator T_n efficient? (We call an unbiased estimator efficient if its variance coincides with the lower bound of the Cramer-Rao inequality).

e) Find MLE (maximum likelihood estimator) of θ^2. Is it unbiased? Consistent? Efficient?

Problem 5

Let $X_1, ..., X_n$ be a sample taken from a $(\theta, 1)$ normal distribution:

a) For testing $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$ find the acceptance region of the significance level α likelihood ratio test.

b) Find the $(1 - \alpha)$ confidence interval that results from inverting the likelihood ratio test of part (a).