Repetition week 44

The mean square error

$$MSE = E\left[(W-\theta)^2\right] = Var[W] + \left(E[W]-\theta\right)^2$$

Score function

$$S(\boldsymbol{X}|\theta) = \frac{\partial}{\partial\theta}\log f(\boldsymbol{X}|\theta)$$

$$E\left[S(\boldsymbol{X}|\theta)\right] = 0$$

$$Var\left[S(\boldsymbol{X}|\theta)\right] = I_{\boldsymbol{X}}(\theta) = -E\left[\frac{\partial}{\partial\theta}S(\boldsymbol{X}|\theta)\right] = -E\left[\frac{\partial^2}{\partial\theta^2}\log f(\boldsymbol{X}|\theta)\right]$$

Let $\tau(\theta) = E\left[W(\boldsymbol{X})\right]$

Cramer-Rao

$$Var\left[W(\boldsymbol{X})\right] \geq \frac{\left(\frac{\partial}{\partial\theta}\tau(\theta)\right)^2}{I_{\boldsymbol{X}}(\theta)}$$

Cramer-Rao iid

$$Var\left[W(\boldsymbol{X})\right] \geq \frac{\left(\frac{\partial}{\partial\theta}\tau(\theta)\right)^2}{nI_X(\theta)}$$

Equality

If and only if $S(\boldsymbol{X}|\theta) = a(\theta)\left[W(\boldsymbol{X}) - b(\theta)\right]$

The multiparameter case

$$Var\left[W(X)\right] \geq \left(\nabla\tau(\theta)\right)^T \left(I(\theta)\right)^{-1} \left(\nabla\tau(\theta)\right).$$

Definition 6.2.21

Let $f(t|\theta)$ be a family of pdfs/pmfs for a statistic $T(\boldsymbol{X})$. The family is complete if

$E_\theta\left[g(T)\right] = 0 \Rightarrow P_\theta\left(g(T) = 0\right) = 1$, for all θ.

Theorem 6.2.25:

Let $X_1, \ldots, X_n$ be iid. from an exponential family i.e.

$$f(x|\boldsymbol{\theta}) = h(x)c(\boldsymbol{\theta})e^{\sum_{i=1}^{k} w(\theta_i)t_i(x)}$$

Then $T(\boldsymbol{X}) = \left(\sum_{i=1}^{n} t_1(X_i), \sum_{i=1}^{n} t_2(X_i), \ldots, \sum_{i=1}^{n} t_k(X_i),\right)$ is complete as long as the parameter space contains an open set in R^n.