Repetition week 45

Sufficiency and Unbiasedness

W unbiased estimator of $\tau(\theta)$.

T a sufficient estimator $E\left[W\middle|T\right]=\tau(\theta)$ and $Var\left[W\middle|T\right]\le Var[W],\ \forall\theta$

T complete $\Rightarrow E\left[W\middle|T\right]$ is the unique best unbiased estimator for $\tau(\theta)$

Hypothesis testing.

$H_0:\theta\in\Omega_0 \qquad H_1:\theta\in\Omega_0^C$

LRT

$$\lambda(\boldsymbol{x})=\frac{\sup\limits_{\Omega_0} L\left(\theta\middle|\boldsymbol{x}\right)}{\sup\limits_{\theta} L\left(\theta\middle|\boldsymbol{x}\right)}=\frac{\sup\limits_{\Omega_0} L\left(\theta\middle|\boldsymbol{x}\right)}{L\left(\hat{\theta}\middle|\boldsymbol{x}\right)}=\lambda^*\left(T(\boldsymbol{x})\right)$$

Reject if $\lambda(\boldsymbol{x})\le c$.

Power function

$\beta(\theta)=P_\theta(X\in R)$

UMP

$\beta(\theta)\ge\beta'(\theta)\ \forall\theta\in\Omega_0$