Chapter 5 Random Sample

Random sample: $X_1,\ldots,X_n$ are iid.

Statistic: $T\left(X_1,\ldots,X_n\right)$

Some properties of Statistics

$X_1,\ldots,X_n$ are $N\left(\mu,\sigma^2\right)$

$\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$ and $S^2=\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i-\bar{X}\right)^2$ are independent

$\bar{X}\sim N\left(\mu,\frac{\sigma^2}{n}\right)$, $\quad \frac{(n-1)S^2}{\sigma^2}\sim\chi^2(n-1)$

T-statistic: $\frac{\bar{X}-\mu}{\frac{S}{\sqrt{n}}}$, In general $T_p=\frac{N(0,1)}{\sqrt{\frac{\chi^2(p)}{p}}}$

$$F_{p,q}\text{ statistic}=\frac{\frac{\chi^2(p)}{p}}{\frac{\chi^2(q)}{q}}$$

$$V\sim\chi^2(q)\Leftrightarrow V\sim\Gamma\left(\frac{q}{2},2\right)$$

$$E\left(V^{-k}\right)=\frac{1}{\Gamma(q/2)2^{\frac{q}{2}}}\int_0^\infty v^{\frac{q}{2}-k-1}e^{-\frac{v}{2}}dv=\frac{\Gamma(\frac{q}{2}-k)}{\Gamma(\frac{q}{2})2^k},$$

$$X_i \sim \chi^2(p_i) \Rightarrow \sum_{i=1}^{n} X_i \sim \chi^2\left(\sum_{i=1}^{n} p_i\right)$$

Convergence concepts

Convergence in probability:

$\{X_i\}_{i=1}^{\infty} \xrightarrow{P} X$ if $\forall\ \varepsilon > 0,\ \lim_{n\to\infty} P\left(\left|X_n - X\right| \geq \varepsilon\right) = 0.$

Weak law of large numbers

$\{X_i\}_{i=1}^{\infty}\, iid,\ \mathrm{E}\left[X_i\right] = \mu$ and $\mathrm{Var}\left(X_i\right) = \sigma^2 < \infty$. Then $\lim_{n\to\infty} P\left(\left|\bar{X}_n - \mu\right| < \varepsilon\right) = 1$

$\{X_i\}_{i=1}^{\infty} \xrightarrow{P} X$ then $\left\{h\left(X_i\right)\right\}_{i=1}^{\infty} \xrightarrow{P} h\left(X\right)$ if h is continuous.