Some repetition from week 42

Sufficient statistics

A statistic $T(\boldsymbol{X})$ is a sufficient statistic for $\boldsymbol{\theta}$ if the conditional distribution of the sample $\boldsymbol{X}$ given the value of $T(\boldsymbol{X})$ does not depend on $\boldsymbol{\theta}$.

A sufficient statistics for a parameter (-vector) $\boldsymbol{\theta}$ is a statistic that in a certain sense, captures all the information about $\boldsymbol{\theta}$ in the sample.

Theorem 6.2.2

If $p(\boldsymbol{x}|\boldsymbol{\theta})$ is the pdf/pmf of $\boldsymbol{X}$ and $q(t|\boldsymbol{\theta})$ is the pdf/pmf of $T(\boldsymbol{X})$, then $T(\boldsymbol{X})$ is a sufficient statistics for $\boldsymbol{\theta}$ if, for every $\boldsymbol{x}$ in the sample space the ratio $\dfrac{p(\boldsymbol{x}|\boldsymbol{\theta})}{q(T(\boldsymbol{x})|\boldsymbol{\theta})}$ is a constant as a function of $\boldsymbol{\theta}$.

Theorem 6.2.6

Let $f(\boldsymbol{x}|\theta)$ be the joint pdf/pmf for a sample $\boldsymbol{X}$. $T(\boldsymbol{X})$ is a sufficient statistics for θ if and only if for all $\boldsymbol{x}$ and all θ.

$$f(\boldsymbol{x}|\theta) = g(T(\boldsymbol{X}|\theta))h(\boldsymbol{x})$$

Minimal sufficient.

Definition 6.2.11. A sufficient statistics $T(\boldsymbol{X})$ is called a minimal sufficient statistics if for any other sufficient statistics $T^{'}(\boldsymbol{X})$, $T(\boldsymbol{X})$ is a function of $T(\boldsymbol{X})$.

Theorem 6.2.3

Let $f(\boldsymbol{x}|\theta)$ be the joint pdf/pmf for a sample $\boldsymbol{X}$. Suppose there exists a $T(\boldsymbol{X})$ such that for every $\boldsymbol{x}$ and every $\boldsymbol{y}$, $f(\boldsymbol{x}|\theta)/f(\boldsymbol{y}|\theta)$ is a constant as a function of $\theta \Leftrightarrow T(\boldsymbol{X})=T(\boldsymbol{Y})$. Then $T(\boldsymbol{X})$ is a minimal sufficient statistics for θ.

Definition 6.2.21

Let $f(t|\theta)$ be a family of pdfs/pmfs for a statistic $T(\boldsymbol{X})$. The family is complete if

$E_\theta\left[g(T)\right]=0 \Rightarrow P_\theta\left(g(T)=0\right)=1$, for all θ.